Optimal Control of Robotic Wheelchair

SE 514 Term Project

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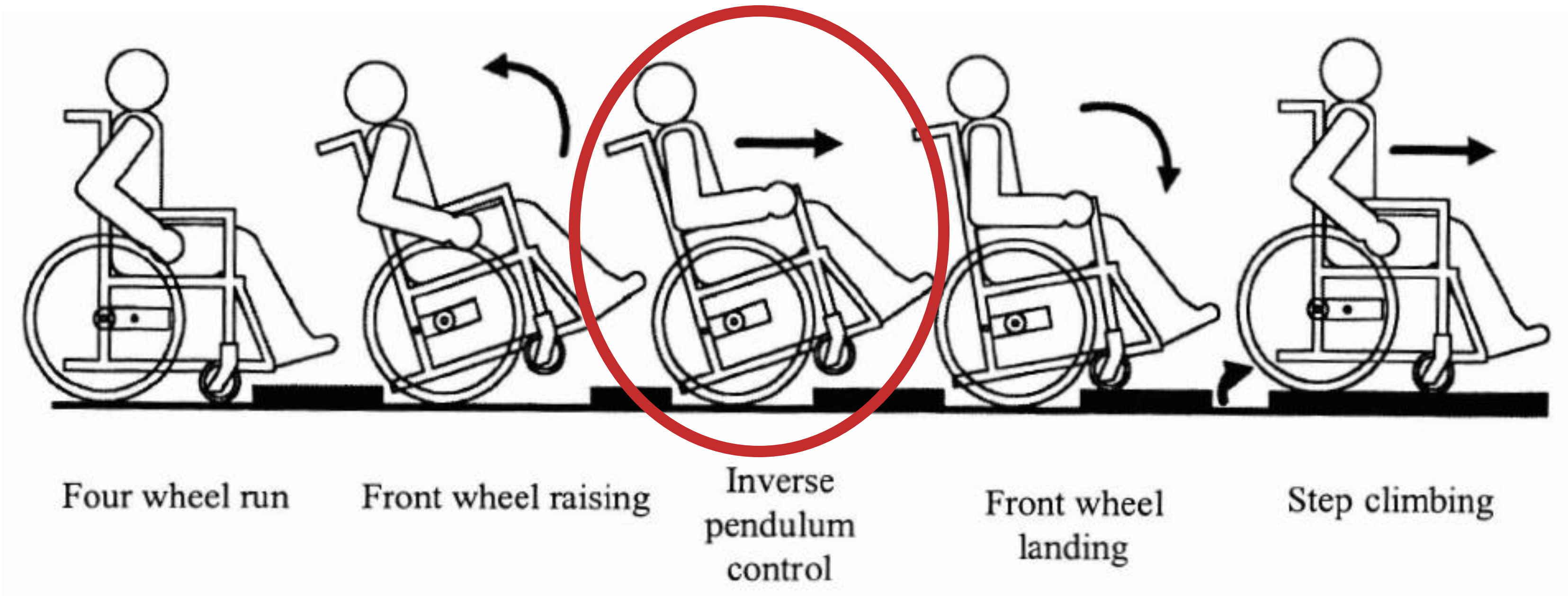
Outline

- **Project Overview**
  - Original Work
  - Physical Description
- **Model Discussion**
  - State-Space Model
- **Control & Results**
- **Comments**

Project Overview

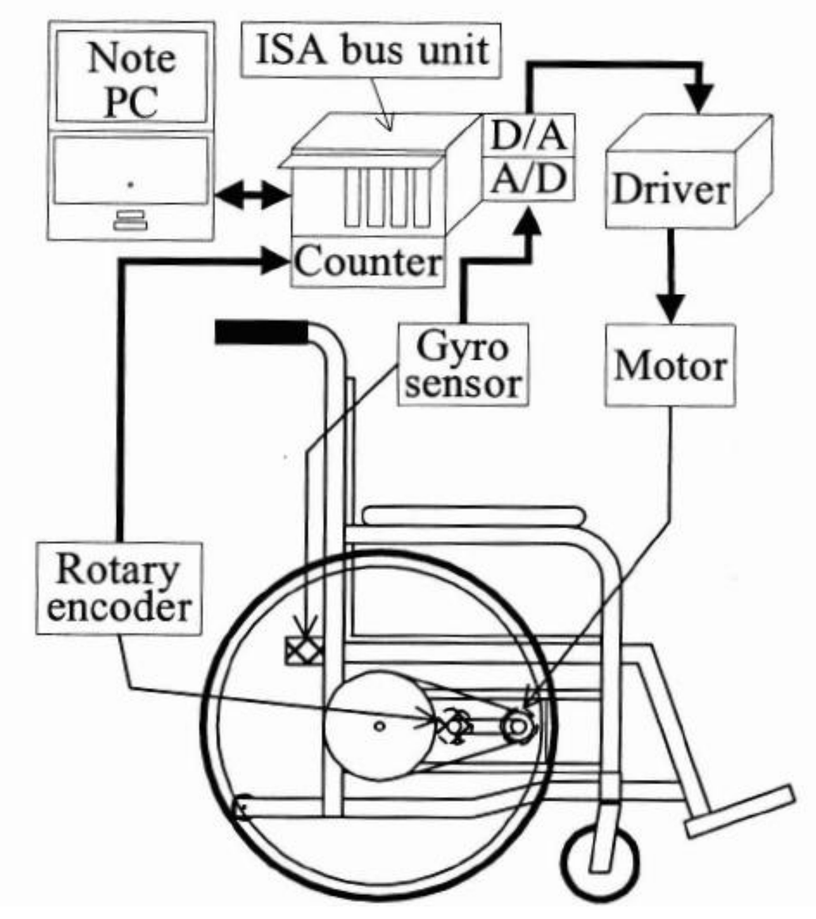
- Authors
  - Yoshihiko Takahashi, Kanagawa Institute of Technology
  - Otsushiro Tsubouchi, Kanagawa Institute of Technology
- Idea



Project Overview

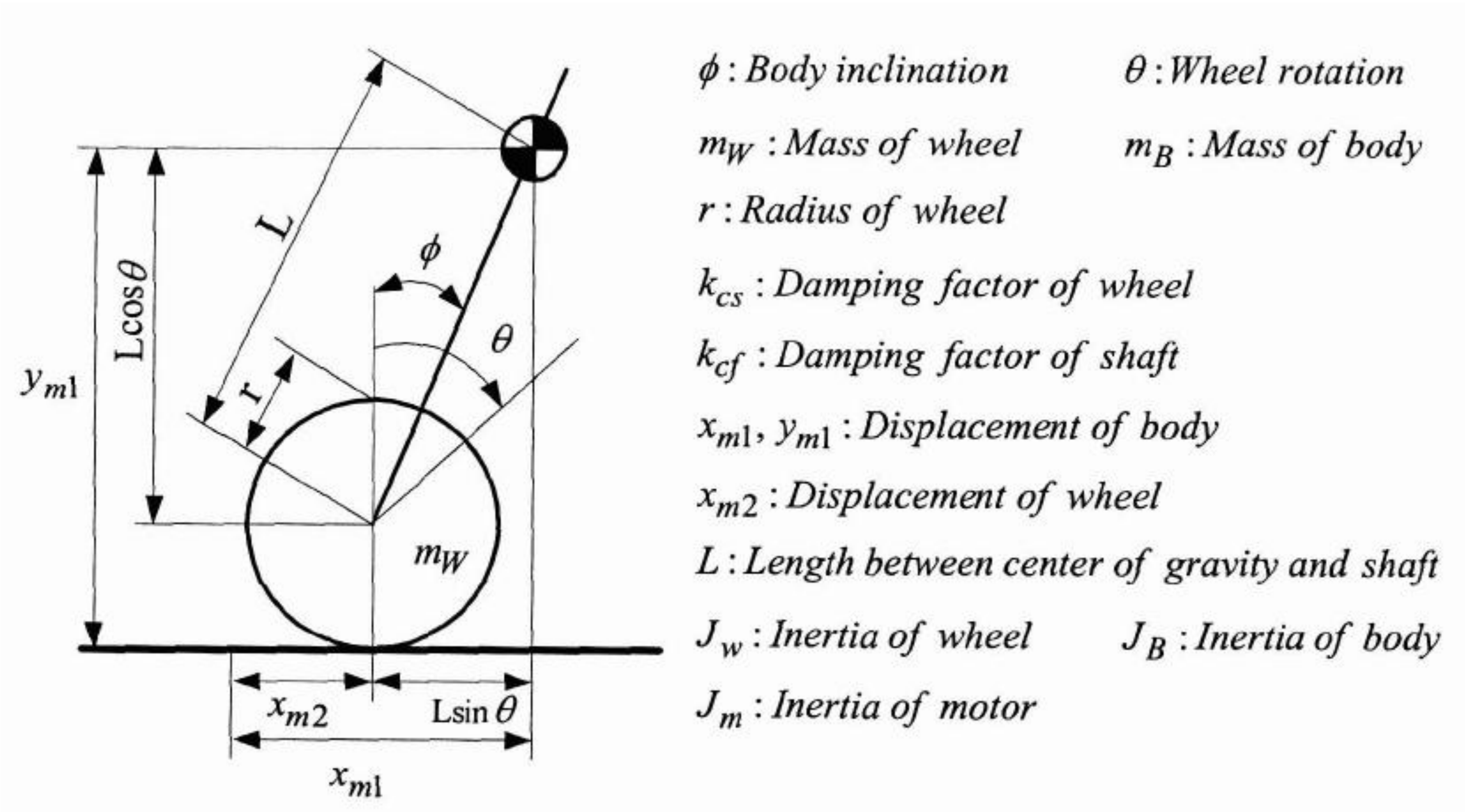
- Hardware Operation

1. Gyro Sensor: Inclination velocity
2. Inclination velocity integrated in PC to give wheelchair inclination
3. Error = Inclination – desired value
4. Error go to Controller
5. Controller output go to DC Motor



Model Discussion

- Wheelchair Dynamics

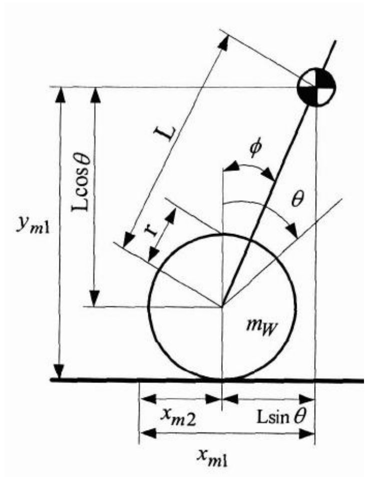


Model Discussion

- Linearization:
  - Inverted Pendulum: around $\phi = 0, \dot{\phi} = 0$

$\Rightarrow$ sin(ϕ)= ϕ

$\Rightarrow$ cos(ϕ)=1



Model Discussion

- Differential Equations

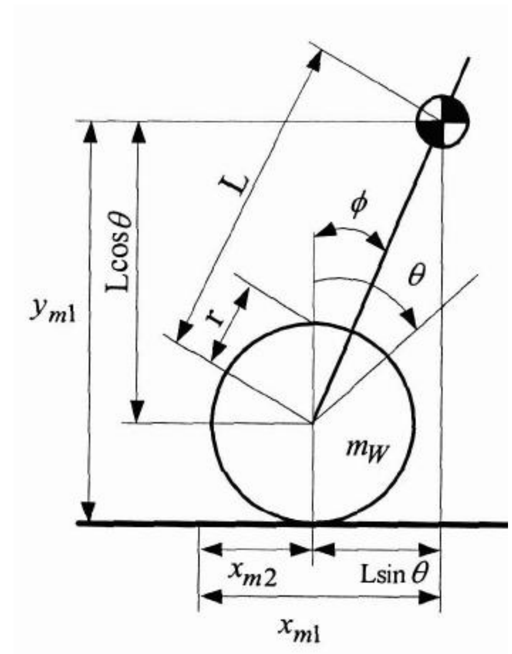
$$\left(M_b L^2 + J_b + J_m K_g^2\right)\ddot{\phi} + \left(M_b r L - J_m K_g^2\right)\ddot{\theta} + K_{cs}\left(\dot{\phi} - \dot{\theta}\right) - M_b\, g L\, \phi = -\frac{K_g K_t}{R} v$$

$$\left(M_b r L - J_m K_g^2\right)\ddot{\phi} + \left\{(M_w + M_b) r^2 + J_W + J_M K_G^2\right\}\ddot{\theta} - K_{cs}\,\dot{\phi} + \left(K_{cs} + K_{cf}\right)\dot{\theta} = \frac{K_g K_t}{R} v$$

  - ϕ: Inclination of chair (rad)
  - θ: Wheels rotation (rad)
  - M_b: Mass of chair & person (kg)
  - L: length between wheelchair shaft & center of gravity (m)
  - J_b: Inertia of chair & person (kg.m^2)
  - J_m: Inertia of DC motor (kg.m^2)
  - K_g: Gear Ratio
  - r: wheel radius (m)
  - K_{cs}: wheel shaft damping (N.m/(rad/sec))
  - K_t: Torque constant of motor (N.m/A)
  - J_w: Inertia of wheels (kg.m^2)
  - M_w: Mass of wheels (kg)
  - K_{cf}: Damping between floor & wheels (N.m/(rad/sec))

Model Discussion

- M_b = 84.16 kg
- L = 0.29 m
- J_b = 29.3 kg.m^2
- J_m = 7.0 x 10^-6 kg.m^2
- K_g = 772
- r = 0.305 m
- K_{cs} = 12.3 N.m/(rad/sec)
- K_t = 0.0239 N.m/A
- J_w = 0.11 kg.m^2
- M_w = 6.52 kg
- K_{cf} = 8.78 N.m/(rad/sec)
- R = 0.84 ohm



Model Discussion

- State-Space Model

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \ddot{\phi}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 6.0236 & 0 & -0.3895 & 0.4464\\ -1.5498 & 0 & 1.0674 & -1.7724\end{bmatrix}\begin{bmatrix}\phi\\ \theta\\ \dot{\phi}\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -0.6955\\ 1.9061\end{bmatrix} v + \begin{bmatrix}0.09\\ 0.025\\ 0.075\\ 0.05\end{bmatrix} w$$

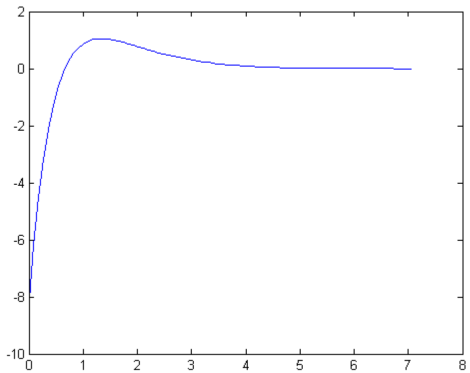
$$\begin{bmatrix}\phi - \theta\\ \dot{\phi}\end{bmatrix} = \begin{bmatrix}1 & -1 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\phi\\ \theta\\ \dot{\phi}\\ \dot{\theta}\end{bmatrix}$$

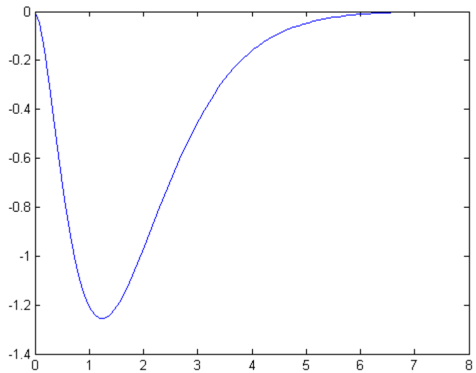
$$z = \phi = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\phi\\ \theta\\ \dot{\phi}\\ \dot{\theta}\end{bmatrix}$$

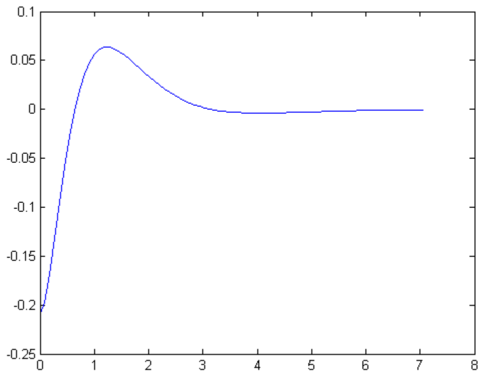
$$x_0 = \begin{bmatrix}-0.21\\ 0\\ 0\\ 0\end{bmatrix} rad \Leftrightarrow \phi_0 = -12^\circ$$

LQR

- 1) $$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, R = 1$$





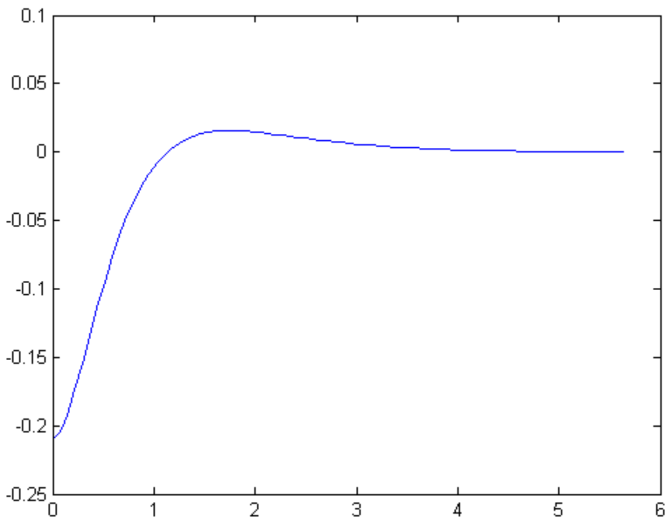


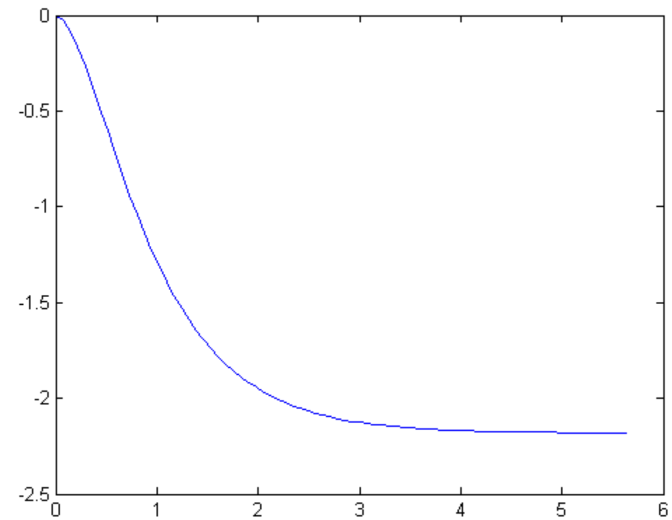
- Increasing weight on ϕ have no effect

LQR

- 2)

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, R = 1$$



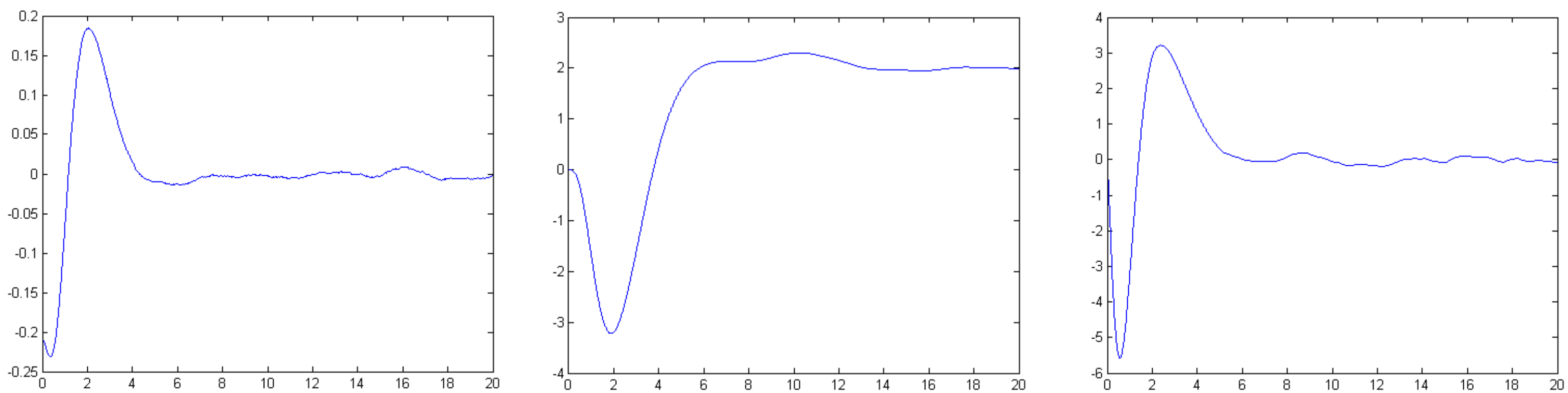


- Little overshoot in ϕ
- No need for Wheel rotation θ to be zero!
- somehow faster settling

LQG

- 1)

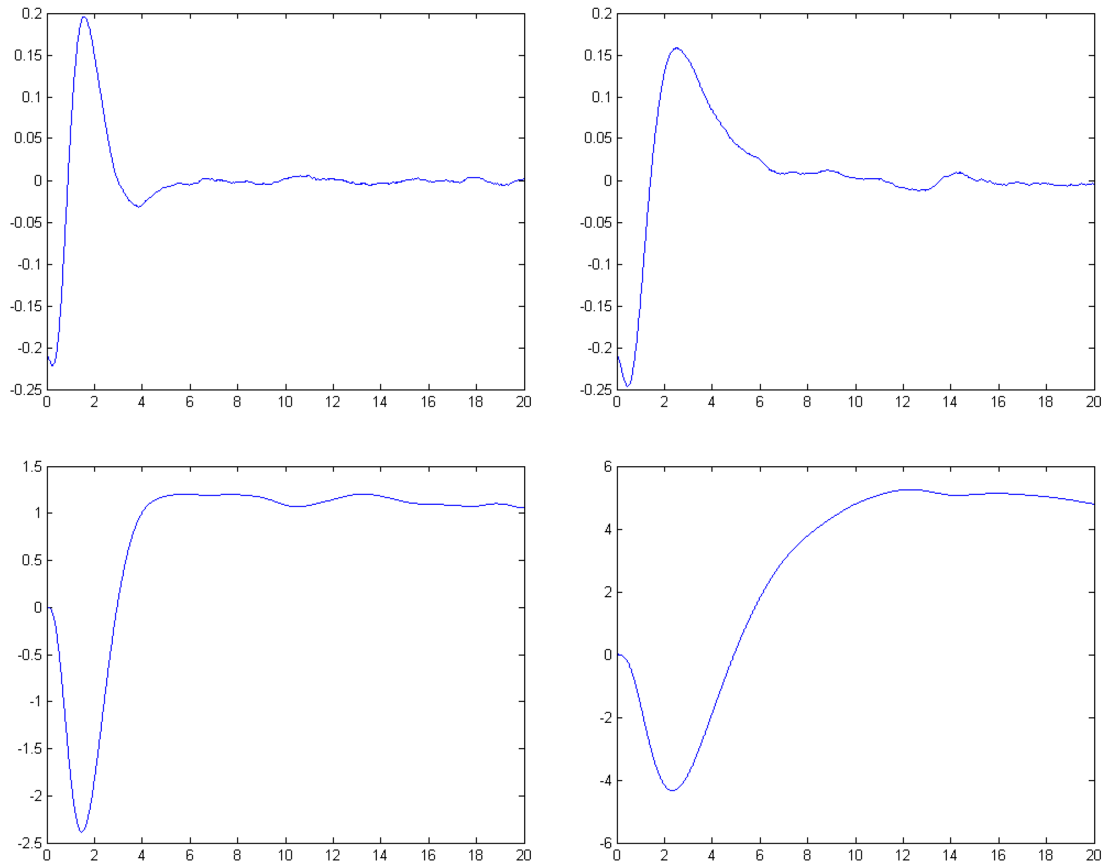
$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, R = 1$$



- Relatively high overshoot in ϕ
- Wheel rotation needs time to back to zero

LQG

- 2) $$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, R = 0.1\,,10$$

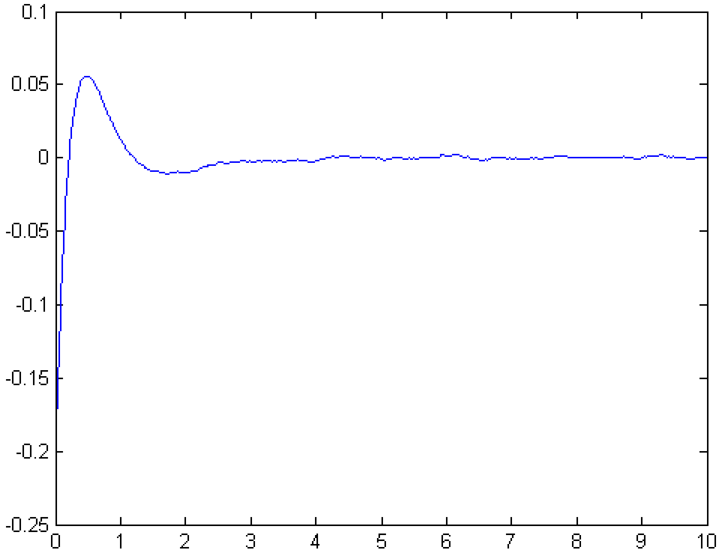


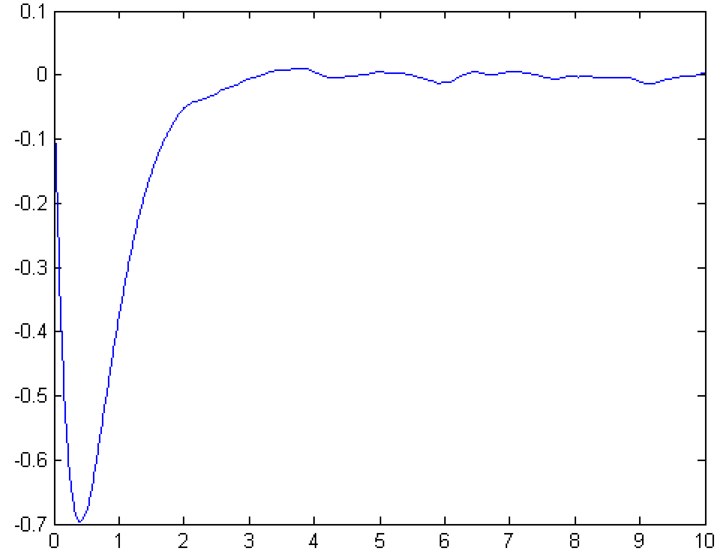
- Decreasing weight on input do not change much

- increasing improves performance with smaller overshoot

- more weight on input ⇒ acceptable current values ⇒ but longer settling time

H2 Control

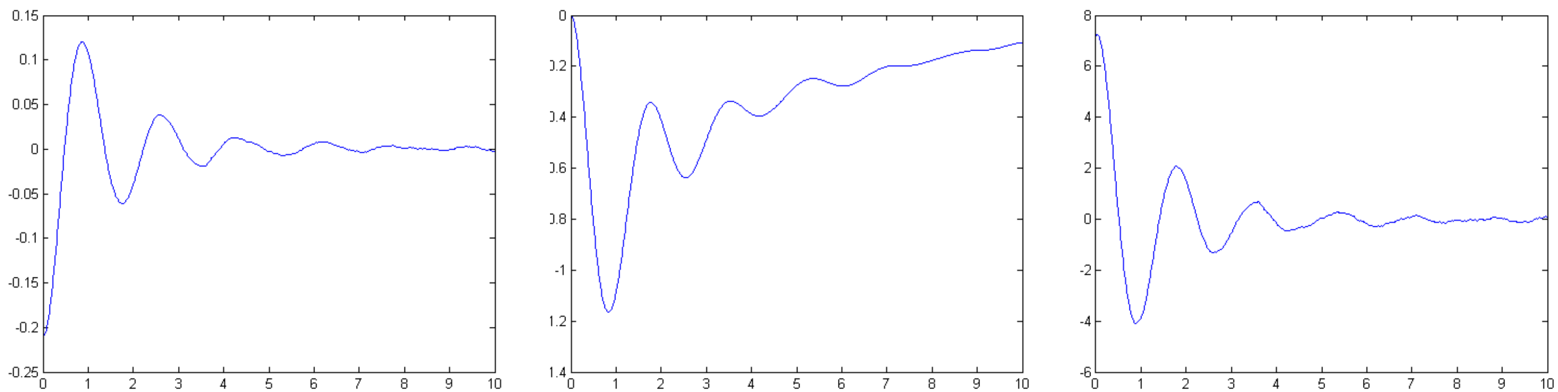




- Faster response & smaller overshoot
- No control input constraint ⇒ huge input jump (~2.5x10^7 A)
- So, boundary on minimization of H2-norm:
  - No change in responses
  - No change in control input behavior

H_∞ Control

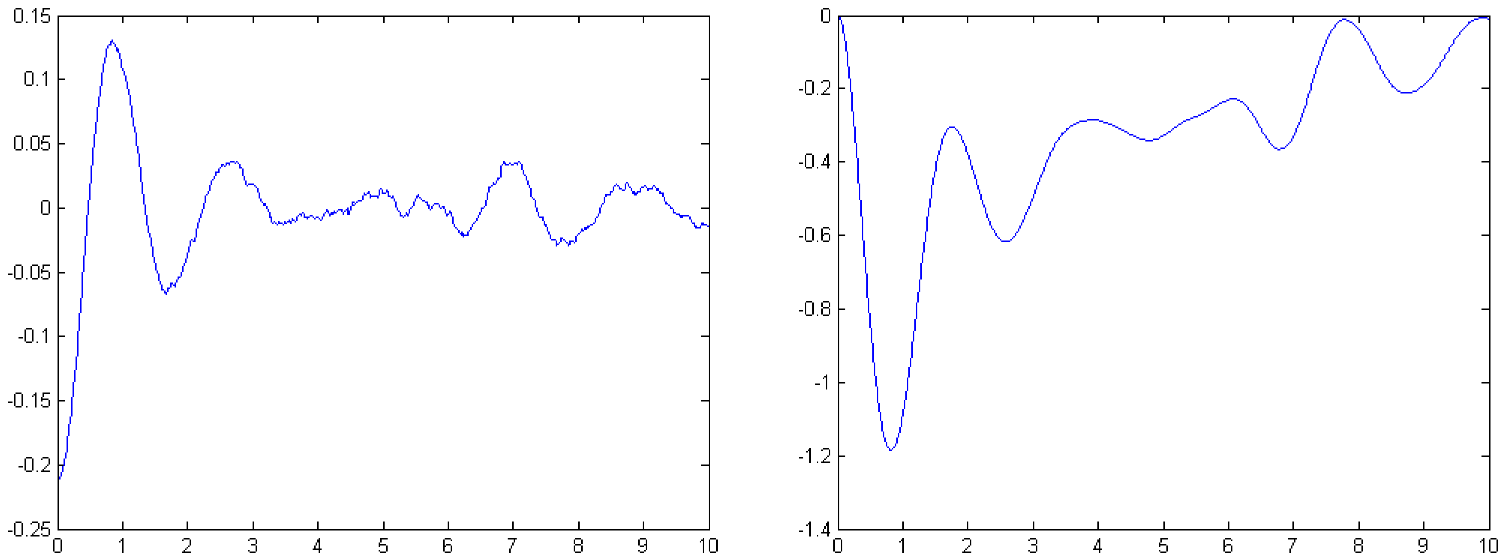
- Same problem for control input
- But trying to minimize γ subject to $\gamma > 0.5$



- oscillatory behavior on ϕ

H_∞ Control

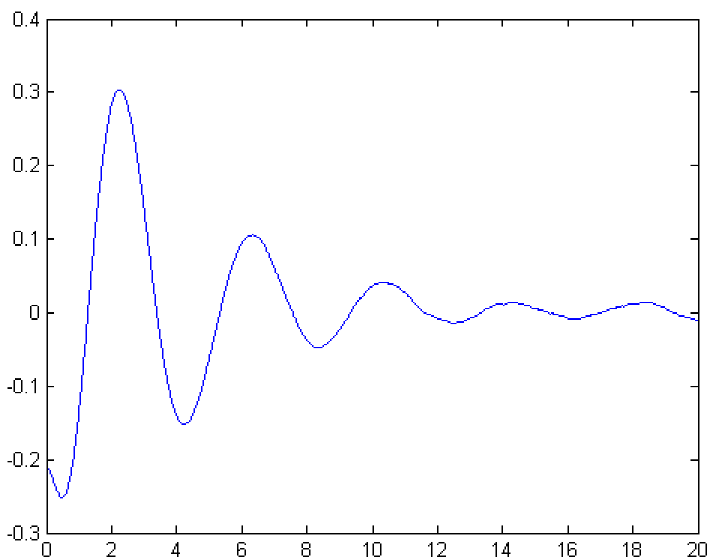
- Making noise 5 times bigger



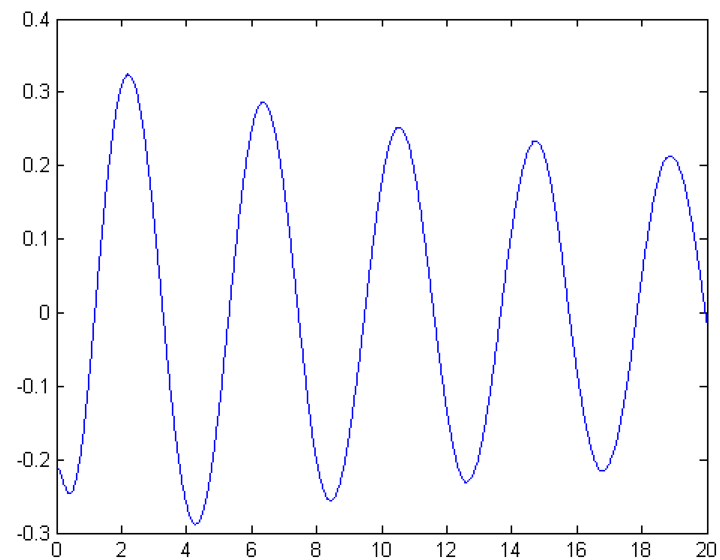
Changes in Physical System

- Using the same LQG design and resulting K
- But:

Wheelchair & body mass increased by 20%



wheel radius 31 to 40 cm



General Comments

- Best practical & acceptable performance:
  - LQG: R=10 (or more) and unit weight on ϕ, θ
  - more weight on input $\Rightarrow$ acceptable current values $\Rightarrow$ but longer settling time
  - Acceptable overshoot (~0.15 rad), settling time (~5 sec) and control input behavior (maximum ~2.5 A)

- In general, inverted pendulum control is OK when ϕ is almost zero. So, big inclinations will need other ways of control

End of Presentation

- Questions R Welcomed