Cooperative Control of Multi-Vehicle Systems

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Mars Exploration

How to control each one? ....... Command Center? Local Decision?

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Automated Transportation Systems

How to arrange vehicles?... Straight line? Triangular?
Outline

1. Motivation
2. Definitions: Cooperative Control
   1. Consensus Problem
   2. Formation Control
   3. Other: Rendezvous, Flocking, Swarming
3. Decentralized Control
4. Connectivity Graphs
5. Control Design
6. Research Directions
7. Simulation Examples

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References


Motivation

• Distributed Operation
• Multi Agents: vehicles, manipulators, ants!
• Local Control: Local objectives, independent
• Limited Capability: Sensing range, Communication, Processing
• Global Objectives: Group goal, Higher-level command
• Complexity: too many agents
• Scalability: large and not fixed number of interconnected systems
• Dynamic: reactive and instantaneous actions
Cooperative Control, **Definition**

- is concerned with **engineered systems** that can be characterized as:
  1. *collection* of
  2. *interconnected* decision-making components (systems)
  3. with *limited processing* capabilities,
  4. *locally sensed information* and *limited inter-component communications*,
  5. all seeking to achieve a *collective (global) objective*

- ‘multi-agent control’, ‘distributed control’, ‘networked control’, ‘swarms’ or ‘coordinated control’ = Cooperative Control

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Consensus Problem

• “the group to reach a position (or agreement) as whole”

\[
\lim_{t \to \infty} x_i(t) = \lim_{t \to \infty} x_j(t) \tag{1}
\]

\[x_i \quad \text{Position of vehicle } i\]

• This behavior can be defined by

\[
\dot{x}_i = \sum a_{ij} \cdot (x_i - x_j)
\]

– averaging
Formation Control

• “to drive the agents to special configuration such that their relative position satisfy a desired spatial and physical constraints”

\[ \dot{x}_i = \sum (x_i - x_j - r_{ij}) \]

• Or in general

\[ \dot{x}_i = \sum g(x_i - x_j) \]
Other concepts

• **Swarms**
  – “to move or gather in group”
  – a specific formation (shape) is not maintained
  – ‘swarm’ often used for *natural biological systems*

• **Rendezvous**
  – “to meet at a pre-arranged position and time”
  – when an agent is approaching its destination faster than other, it tries *dynamically* to slow down to meet the requirement of the group in position and time (**Military** application)

• **Flocking**, Rules of *Reynolds*:
  – to stay close to neighboring flock mates (Cohesion);
  – to avoid collision with neighboring flock mates (Separation);
  – to match velocity of neighboring flock mates (Alignment).
Decentralized Control

• Decisions (control) of each individual agent is computed *locally*
  – No Central center of control
  – No coupling

• Decentralized Controller:
  – properly designed to meet the objective of the whole group
  – cooperation needs sharing information, through *communication*

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Connectivity Graphs

- Utilize *Graph Theory*
  - Assuming access to neighbor’s state (position), only neighbors!
  - Vehicle 1 can ‘see’ 2, 4 (so, neighbors) & vehicle 3 ‘see’ 4, 5, 6
  - This connectivity graph is characterized by **Laplacian Matrix**

\[
L = \begin{bmatrix}
1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\
0 & 0 & 0 & 1 & -1 & 0 \\
-\frac{1}{3} & 0 & 0 & -\frac{1}{3} & 1 & -\frac{1}{3} \\
0 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 1 \\
\end{bmatrix}
\]
Connectivity Graphs

- Recall

\[
\dot{x}_i = \sum_{\forall j \in \mathcal{N}_i} a_{ij} \cdot (x_i - x_j)
\]

- So for a group of \( N \) vehicles,

\[
\dot{X} = -LX
\]

\[
X = [x_1 \ x_2 \ \cdots \ x_N]^T,
\]
Convergence (Stability)

- You can check the eigenvalues of 
  
  \[-L.\]

- \(L\) could be \textit{time-varying} \(\text{(because of communication losses)}\)

- A \textit{connected} graph is when there is a \textit{path} between any two agents
  
  o For \textit{undirected} graph (i.e. edges of a graph has no direction, both vehicles see the other)

  - “\textit{Consensus is reached asymptotically if there exist infinitely many consecutive bounded time intervals such that the union of the graphs over such intervals is totally connected}”, Jadbabaie \textit{et al.}

- Consensus point (steady state) = average of initial positions

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Control Design (decentralized)

- Performance Output

\[ z_i = G(x_i) = \sum_{\forall j \in N_i} g(x_i - x_j) \]

Interaction Function

- One proposed design

\[ u_i = -K \sum_{\forall j \in N_i} a_{ij} \cdot (x_i - x_j) \]

- Other design

\[ u_i = -\sum_{\forall j \in N_i} g(x_i - x_j - d_{ij}) \cdot (x_i - x_j - d_{ij}) \]

- \( g(.) \) could be designed from the concept of Artificial Potential Force,
- Provide collision avoidance also

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Research Directions

• **Cooperative State Estimation**
  – accessibility to other vehicles state, *not necessarily!*
  – existence of noise
  – Estimating \((x_i - x_j)\) \((x_i \text{ not required})\)

• **Hybrid Control:** *Control Theory & Computing*
  – non-smooth dynamics (discontinuous)
  – for example: all of a sudden new vehicle join the group, \(N\) changed

• **Stochastic Approaches**
  – To *model* change of connectivity graphs

• **Time-delay Systems**
  – delays in information exchange between vehicles

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Simulation

- Consider 2 connectivity graphs

\[
L = \begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix}
\]
Simulation Result

- 3 vehicles, 2D motion with single-integrator (i.e. 1\textsuperscript{st} order) system.
  \[ \dot{X} = -LX \]
Cooperative Control, Ending Remarks

- Relatively new field of research
- Bio-inspired
- Wide and interesting applications
- Decentralized Solution
- Graph theory & Control Theory
THANKS

• Q & A

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