# Cooperative Control of Multi-Vehicle Systems

Mohammad Shahab, ID227598, *Systems Engineering Department, KFUPM, Dhahran, Saudi Arabia*

***Abstract*—This paper will review a relatively new field of research, namely the area of *Cooperative Control*. This paper/report will cover many aspects about the field of cooperative control. The paper can be considered as a comprehensive study of many, if not all, related elements of cooperative control. This field studies the interconnections between systems to act collectively towards global objective or more. The study in this paper is induced by *control theory* being the steering wheel. In this paper, along with the detailed review of fields of cooperative control, a specific area of application is set as the regular example. The paper will consider the area of *Multi-Vehicle Autonomous Systems*. Decentralized control solutions can be employed in local systems and giving results matching global objectives. The powerful utilization of *Graph Theory* gives solid mathematical foundation to model communication interconnections. Many techniques under the field of cooperative control have interesting findings. The main principle of *Consensus* was set as the standard problem that included most of cooperation problems. Special tasks as *formation control* was discussed with detail in relation with other aspects in control theory. Applications and research status were discussed to give many possible areas of practical commercial use.**

***Index Terms*—Cooperative Control, Decentralized Control, Consensus Problem, Formation Control, Multi-Vehicle Systems, Graph Theory, Swarms,**

## I. Introduction

CONTROL THEORY has always been related to many research disciplines. Since many years, conventional and modern control theory techniques were studied and developed. Applications evolved a lot throughout the years. As applications span grows more and more with different needs, research move toward new horizons. With the current technology power, utilizing new techniques and methods became easier. Specifically, computing and communication technologies gave more solid and simple handling of problems. With the growing complexity of many problems nowadays, new approaches are needed to tackle these complexities. Complexity of problems comes in the form of difficult analysis or design. However, with the current research resources and skills, development is rapidly growing.

This paper will review a relatively new field of research, namely the area of *Cooperative Control*. This paper/report will cover many aspects about the field of cooperative control. You can think of the paper as a comprehensive survey of many, if not all, related elements of cooperative control. This field studies the interconnections between systems to act collectively towards global objective or more. The study in this paper is induced by *control theory* being the steering wheel. Other approaches can be found by a lot of researchers in the literature. In this paper, along with the detailed review of the field of cooperative control, a specific area of application is set as the regular example. The paper will consider the area of *Multi-Vehicle Autonomous Systems*. This application will be as the main example, but this will never be, in the paper, the only example studied and analyzed. The paper/report is divided into two overlapping major parts, one is background and the other is mathematical development. This is a rough partitioning of the paper.

This report/paper will start to give the essential background about the field of Cooperative Control. In Section II, the paper will review, in detail, the common descriptions and motivations found in literature along with the basic definitions and goals of Cooperative Control with relation to other approaches found in the research community. Section III will give the detailed applications and areas of interest of Cooperative Control. Then, mathematical details of the paper will begin. So, needed background knowledge for the reader will be presented in Section IV, namely *Graph Theory* and *Multi-Vehicle Dynamic Systems* problem formulation. The study of different *Models* describing cooperative systems with *Stability Analysis* will be given in Section V, with providing various techniques and approaches. Section VI will offer diverse *Control Design* techniques. Section VII will investigate the current status of research around the world with giving glimpses of what could be future research directions in the field of cooperative control. Simulation examples are provided in Section VIII. Section IX will conclude the report/paper.

## II. Cooperative Control Background

In this section of the paper, definitions and motivation of the field is provided. II-A is reserved for what is motivating research in the field. II-B will give different definitions and terminologies found in literature concerning cooperative control. II-B will be the most important part in Section II because of the detailed and informative nature of it. II-C provides the reader with other approaches that deal with the area.

Mohammad Shahab is with Systems Engineering Department, King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia. He belongs to the department as a research assistant pursuing for master degree. (Website: http://staff.kfupm.edu.sa/se/mshahab - email: g200275980@kfupm.edu.sa).

### *A. Motivation*

Recent technology provides advanced performance and capability with more efficient computation and less expensive communication. With growing need of *distributed* operations in several applications, complexity of problems increases. 'Distributed' is in the sense of the having different components (or *agents*) with *local* objectives towards fulfilling a global objective or more. This view can be imagined with association with many areas. The aspect of *Autonomy* is essential to have machine decision-making systems. Local operation can be related to the dynamic nature of agents. Local objectives are met using well-made techniques available. 'Local agents' can be mechanical manipulators in an assembly line, multiple mobile robots landed on Mars or even a herd of animals in the wild. Other detailed information about application will be discussed in III. Every 'agent' is concerned with some kind of individual purpose or goal. However, when having a group of agents, 'coordination' or 'cooperation' should be maintained. The group when acting towards meeting a global goal, 'information' should be shared in order to have the accurate coordination and cooperation.

**Cooperative control** is concerned with engineered systems that can be characterized as a 1) collection of 2) interconnected decision-making components (systems) with 3) limited processing capabilities, 4) locally sensed information and limited inter-component communications, 5) all seeking to achieve a collective (global) objective. Global (opposite to local) processing and communication with big number of agents make the problem more and more complex. An approach to only operate locally to meet global goals is adopted. This 'local' strategy makes life easier with the current communication/control capabilities. As mentioned before, complexity could be decreased by proper design of the agents.

Other issue that motivates this field of study can be the need of *scalability*, i.e. large and not fixed number of interconnected systems. Another big motivation is to implement the *dynamic systems and control theory* approaches into the design of agents. The 'dynamic' element is of big importance because it provides the solution for giving *reactive* and *instantaneous* actions towards *robust* global outputs. Motivation also comes also from examples of 'successful' cooperative systems found in *nature*. Examples of cooperative systems in nature can be schools of fish, flocks of birds or even bacterial colonies. These existing systems can give us a look upon how to mimic the robust and dynamic behavior of them. On the other hand, one can consider that a common problem with a group of systems, in general, is the environment surrounding these systems. The environment is dynamically changing and *uncertain*.

### *B. Definitions*

Here in this part of the paper, detailed information will be presented for the reader. Unlike many papers found in literature, this paper will provide, hopefully, different points of view towards the problem of cooperative control. Having that said, most papers deal with the problem with usually changing terms and notions depending on the application. However, generally all problems come from the same issue of the need of cooperation. Actually, most researchers in the field flip their language back and forth. At the end, they all serve for the same problem.

In section II-A-1of the report, a start of introducing several researchers around the world with research areas that have done work related to cooperative control. In II-A-2, concepts inside the field of cooperative control will be discussed in detail.

#### *1) Related Research*

Cooperative control can be found and viewed in the same context with relation to *Networked Systems & Control*, which studied by prominent researchers as R. Murray and Olfati-Saber (e.g. as in [1]). The study of cooperative control of multi-agent systems in relation with *Networked Dynamical Systems* and *Graph Theory* can be seen in the work of Jadbabaie (e.g. as in [10]). Relation to *Motion Coordination & Planning* is studied by researchers as V.J. Kumar. Also, relation with the study of *Mobile Sensor Networks* appears in the work of Naomi Leonard. Relation to Motion Planning for *Robotic Networks* is seen in the research of Francesco Bullo and Raffaello D'Andrea. *Multi-agent Control & Estimation* research can be seen in the work of Domitilla Del Vecchio.

Other point of view is provoked by natural existing cooperative systems. Observing and study of animal collective behavior lead to the field of *Swarms* or *Swarm Robotics*. Even the word 'swarm' was used very often on parallel with other terminologies. Early study of *Swarm Stability* is done by Passino [3]. Also, biologists have contributed to the study of animal collective motion as in the work of Vicsek and Reynolds. *Self-organizing Systems* study is related to cooperative control in the sense of the objective of a group to achieve a global goal as found in the work of Kalvins.

#### *2) Cooperative Control Concepts*

As observed before, terms of 'multi-agent control', 'distributed control', 'networked control', 'swarm' or 'coordinated control' are all equivalent to *Cooperative Control*. Under that field, there are many hot research areas. Here in this section, we will give information about what research is done in the name of cooperative control.

**First**, a concept should be cleared. As cooperation needs sharing information, techniques should be designed in an efficient way in order to reduce complexity and improve practicality. As mentioned before, decisions of each individual agent is computed locally. This concept is called *Decentralized Control* as opposed to centralized control. In a centralized solution, there is one decision-maker that has access to all information incoming from all agents. Furthermore, there is typically a communication cost in distributing gathered information. As a central 'controller' is responsible for the whole problem, computational complexity increases because different reasons like the size (number of agents). Also, communication complexity increases in a centralized control because the limits of bandwidth and connectivity. Actually, communication failure in the central controller means the failure of the whole system even if not all agents have troubles.

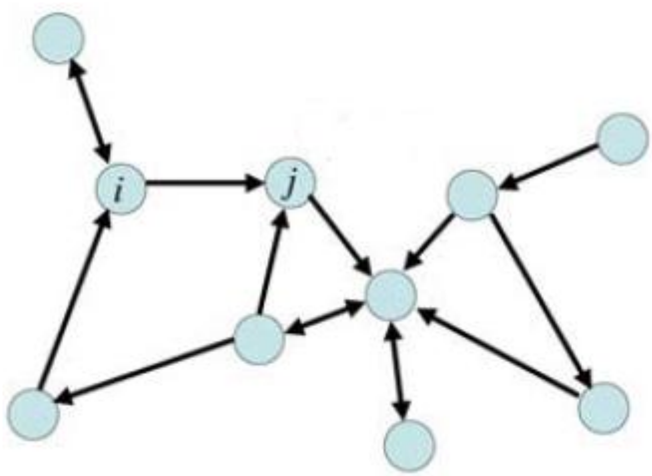


Fig. 1. Graph of the Agents. The graph defines the interconnections (communication) between agents. Graph nodes being the agents and edges being the interconnections.

**Decentralized Control** has the advantage of not limiting the capabilities of each agent. The reader can imagine considering multiple mobile robots for the purpose of, for example, Mars exploration. Each robot has its own controller that is responsible for local motion and tasks. The same controller is properly designed to also cover for the objective of the whole group. The reader has to put in mind that an agent's controller has no coupling with any other controllers of other agents, i.e. each controller is fully independent. This somehow has similarities with *hierarchal control*.

**Secondly**, under the inspiration of biological systems and the need of technologies, many problems have been defined as the main areas under cooperative control: *Flocking*, *Consensus*, *Rendezvous*, *Formation Control* and *Swarming*. All cooperative control problems contain the analysis of behavior of a group of agents. The agents are arranged in some geometrical map or *Graph* in state space of the agents (e.g. position or heading). This graph defines the interconnections between agents. Figure 1 gives a pictorial view of this concept, graph *nodes* being the agents and *edges* being the interconnections. We assume that the graph changes dynamically. Of course, the graph depends on the states of each agent. A complete introduction to Graph Theory will be delivered in Section IV. Each one of field of study under cooperative control will be explained in some detail.

- *Swarming*: "*to move or gather in group*". In a swarm, a specific formation (shape) is not maintained. Usually, swarms as a topic is studied as any other field under cooperative control. The difference is only that the word 'swarm' often used for natural biological systems or for reasons will be discussed in II-C.

- *Consensus*: "*the group to reach a position as whole*". This definition is applicable to many applications. One can consider Swarming as a part of consensus. The goal in both is similar. In consensus:

$$\lim_{t\to\infty} x_i(t) = \lim_{t\to\infty} x_j(t) \quad (1)$$

  As $x_i$ the position (state) of agent $i$, with $x_j$ being the state of all agents other than $i$. In more general terms, 'consensus' can mean to reach an agreement about the states of all agents, not necessarily the position. But in a normal multi-vehicle system, the general goal of cooperative control is related to the position of each agent. In network community, consensus is defined more for computer network applications rather than mobile vehicles. Consensus is often regarded as consisting of many subfields. Actually, other fields in cooperative control can be treated as 'consensuses'.

- *Flocking*: "*to travel in a flock*". Flocking studies is inspired by biological systems of birds and fish. This field is pioneered by the Reynolds as been showed in [4]. Reynolds introduced three rules, often used by computer animation community. These rules define the behavior of any flock geometrically. The three flocking rules of Reynolds are
  1) Flock Centering: to stay close to neighboring flock mates (Cohesion);
  2) Collision Avoidance: to avoid collision with neighboring flock mates (Separation);
  3) Velocity Matching: to match velocity of neighboring flock mates (Alignment).

  To achieve the 3 above objectives, *Flocking Algorithms* were developed. To fulfill these objectives, an agent in a flock could attain to the motion of a leader or virtual leaders defined depending on the topology (graph) of the agents.

- *Rendezvous*: "*to meet at a pre-arranged position and time*". This is more interesting problem. This is a special case of consensus problem. Here consensus is agreed upon specific position *and* time. All agents must reach the predefined position simultaneously. So even when an agent, for example, is approaching its destination faster than other, the agent tries dynamically to slow down to meet the requirement of the group in position and time. Different speeds of agents should be taken in consideration in the design to fulfill the requirement. Applications of the rendezvous problem can be imagined especially in critical ones, such as military.

- *Formation Control*: according to [5], a Formation Control problem defined as "*to drive the agents to special configuration such that their relative position satisfy a desired topological (spatial) and physical constraints*". In simple words, this means that the required objective is to arrange the agents in desired shape. This is a hot topic of research. Many papers investigate many problems associated with formation control. Actually, formation control is a specific problem under cooperative control world. It is obvious that it is a consensus problem. It also involves some attachments to flocking and rendezvous if required by a specific requirement. Desired formation can be defined in 2D or in 3D. Formations should maintain themselves even in the presence of obstacles. Formation Control will be discussed with more elaboration in Section V & VI.

### C. *Other Approaches*

Here in this small section, another approaches dealing with cooperative systems is discussed. The approaches here are different than what previously discussed due to the background of researchers. Specifically, here the discussion is just giving another perspective of the problem. The perspective is coming from the area of *Artificial Intelligence*. As been referred earlier, Swarms has been studied in the context of intelligence. There is a field standing by itself called *Swarm Intelligence*. The Multi-Agent problems are treated from *behavior* point of view. You can see the great work in the area of *Behavior-Based Robot Cooperation* through the research of Lynne Parker (e.g. as in [11]). However, most of artificial intelligence approaches lack the dynamic and instantaneous actions required by a robust cooperative system.

Other Researchers have work related to cooperative control are many because the multi-disciplinary nature of the area. The collection of names given here is just to inform the reader of world's leading researchers in the field. So to name some people working in cooperative control area or related area: Calin Belta, Andrea Bertozzi, Magnus Egerstedt, Rafael Fierro, Bruce Francis, Jie Lin, Steve Morse, Daniela Rus, Shankar Sastry, Herbert Tanner, Claire Tomlin and George Pappas. This is a short list of prominent researchers working in cooperative control.

## III. Applications of Cooperative Control

Here in this section, a practical point of view is discussed of the field. We will give different feasible practical applications that cooperative control methods can be used. This section is divided into three parts. Section III-A will give closer look into critical applications of Military Systems and Space Systems. Section III-B will shed the light into more civilian applications that touch peoples' daily life. Section III-C will just give 'applications' in research and academia environments. The above partitioning is done for the purpose of making it easier for the reader to track his/ her area of interest.

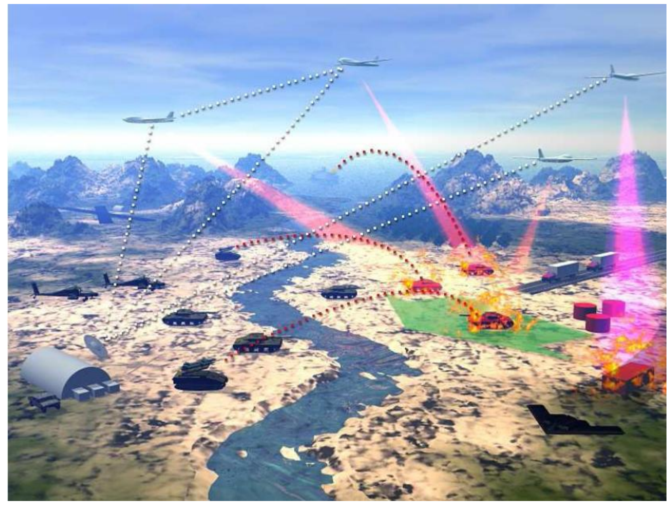

Fig. 2. Example of Application utilizing cooperative control techniques. Unmanned Aerial Vehicles (UAVs) is an area with great research work done. Cooperation and coordination are major issues studied.

### A. *Military & Space Applications*

Battlefield environments are similar to planetary environments. A great deal of sophistication is required to develop systems that can work effectively in those regions as it is depicted in figure 2. With the advanced development of unmanned vehicles, central command centers are no more practical with the emerging need of large number of vehicles and the large land coverage areas. The dynamic nature of the surroundings implies also the need of cooperation.

#### 1) *Space*

Space Exploration application has wide relation to cooperative operations. A mission to Mars needs a collection of mobile robots exploring the planet in parallel with sharing the information together referring to mother center. More applications can be imagined to utilize cooperation in Mars, for example. Other explorations can be employed in our planet, Earth, to look for valuable resources (e.g. oil in the case of Saudi Arabia) in deserted and harsh areas. A cooperation element can add a great deal of ease to the problem.

Another major application in space is Satellites Formation. These formations are applied usually to microsatellites to meet certain objective for the collection resembling one complex satellite [12]. Collaboration is needed from all microsatellites to meet the unified goal, for example imaging.

### 2) *Military*

Here, more detailed look upon military applications is presented. The talk discussed here is obtained from the work of Murray [13]. Research was done heavily in the fields of *Unmanned Aerial Vehicles* (UAVs) and *Unmanned Ground Vehicles* (UGVs). Formations with UAVs are deployed in 3D space. For a critical military mission, a robust formation control must be maintained. For a follower aircraft, tracking the leader is the objective. However, trajectory planning for the leader is an element in the design for the whole group. One problem is to minimize the energy spent in the trajectory while keeping a *rigid* formation.

Fig. 3. A platoon of cars. Automated Transportation Systems is a daily solution for increasingly crowded roads. Cooperative control could give 'smooth' solutions to this.

Cooperative Surveillance is an interesting application. This can be defined by the problem describing the state of a geographic area through the decentralized collection of vehicles. Rendezvous problem in military is important as mentioned before. One application of that is to minimize the exposure to radar via assigning a split strategy with a limited time.

## B. *Civil Applications*

In this section, more people related applications are explored. This is still a new area of investigation. One of the well-studied applications is *Automated Highway Systems* or *Intelligent Transportation Systems*. Forming car *platoons* (line of cars one behind the other with separating distance of one meter for example) is an example of ideas of related system. Automated transportation systems are kind of dreams that could come to reality with proper control design. Imagine fully automated cars with collision-avoidance capability and other features of coordinated motion. Here the information shared by agent (cars) are either coming from communication between them other sensory information such as vision systems, i.e. camera. Benefits of an automated transportation are many. To name few: reducing congestions, increase road safety and of course driverless commuting.

Another application is a civil environment is related to safety and security. One can imagine various applications. *Land mine detection* can use cooperation between vehicles in order to demine the land. Another application that can be investigated is the process of searching for missing people in area hard for humans to operate in; or to search for dangerous materials or bomb in an evacuated building.

## C. *Applications in Academia & Research*

Here, just a brief overlook upon applications of cooperative control in academia and research is presented. One big application is *Mobile Sensor Network*. The main difference from conventional sensor networks is the element of mobility as a part of control. The global goal is to gather information for specific purpose. Cooperative control comes as an aid for agents, here mobile sensors, to maximize the amount of information collected. Some name this *adaptive sampling*.

Another well-developed area of research done in cooperation is the RoboCup competitions. The competition aim is that by 2050, a team of humanoid, i.e. human-like, robots can beat the human football (soccer) world cup winner at that time, playing in the regular rules and environment as standard. This big aim is being chased through an annual competition in different modes of play. Until now, cooperation in robot matches is still investigated from the artificial intelligence point of view. With growing research of cooperative control, more improvements can be added to the current RoboCup technologies.

Labs research also, of course, uses methods of cooperative control to solve many research problems available in the same field or in other areas as well. Research is not only done in engineering departments. Biological and environmental studies study different models of swarming or cooperation in order to have more solid understanding of nature of several kinds of life beings which practice cooperation and coordination in a dynamic and reactive way.

# IV. NEEDED INFORMATION IN COOPERATIVE CONTROL

With this section, the solid *mathematical* study of the field of cooperative control starts. From now on, previous introductory discussion is assumed to be understood by the reader. Terms and notions of cooperative control literature will be used as explained previously. Section IV-A will set the problem formulation of *Multi-Vehicle Systems*. Different dynamic models will be discussed. Cooperative control models and algorithms will be left for Section V. In Section IV-B, preliminary information about Graph Theory will be delivered with the required amount linked to cooperative systems.

### A. Autonomous Multi-Vehicle Systems

Multi-agent systems can be defined, in general, by "*systems that consist of multiple agents or vehicles with several sensors/actuators and the capability to communicate with one another to perform coordinated tasks*". Different dynamic models will be discussed here. All will give the picture of the *local* behavior of the group of agents. Notations in this section will be carried out in the rest of the paper.

Consider a group on *N* vehicles with identical dynamics of

$$\begin{aligned} \dot{x}_i &= q(x_i, u_i) \\ y_i &= h(x_i) \end{aligned} \tag{2}$$

With $i = 1, \ldots, N$ vehicles. With $x_i \in \mathbb{R}^m$ being the original states of the vehicle defined as the position. $u_i \in \mathbb{R}^p$ being the control input to the vehicle. In our case, if we assume ground vehicles, control input can be throttles, for example. With $q(.)$ and $h(.)$ define the state and output equations respectively. With $f$ reflecting the mechanics of the vehicle. In this paper, without loss of generality, $m = 2$ can be assumed. This means that the vehicles at hand are defined with motion in a 2D plan. 3D motion can be defined for UAVs, foe example. Note here that the time index $t$ has been dropped deliberately from the states, outputs and inputs. This will be the case in the rest of the paper unless other thing mentioned.

At this stage, we can assume the accessibility of each vehicle to other vehicles states. Further elaboration about the communication models will be discussed in Section V. Also, here we are going to present two kinds of dynamic models of the vehicles. Both is presented here for the purpose of giving the reader comfortable informative view of systems under study. One will describe a two-wheeled vehicle. The other will describe a linear vehicle model.

#### 1) Wheeled Vehicle Dynamic Model

The reason of giving the dynamic model of this kind of vehicle is to shed the light in a common type of vehicles studied in research. The dynamics of this vehicle, as described in [6[, can be as

$$\begin{aligned} \dot{x}_i^x &= v_i \cos x_i^\theta \\ \dot{x}_i^y &= v_i \sin x_i^\theta \\ \dot{x}_i^\theta &= \omega_i \end{aligned} \tag{3}$$

With $x_i^x, x_i^y, x_i^\theta$ being x-position, y-position and heading of the vehicle respectively. With $v_i \; and \; \omega_i$ being linear and angular velocities respectively as the control inputs of the system. From (2), $x_i = \begin{bmatrix} x_i^x & x_i^y & x_i^\theta \end{bmatrix}^T$ and $u_i = \begin{bmatrix} v_i & \omega_i \end{bmatrix}^T$. It is obvious that this dynamic model is nonlinear. For some *i* and $j$ vehicles, the distance $l_{ij}$ between the two is computed by $l_{ij} = \sqrt{\left(x_i^x - x_j^x\right)^2 + \left(x_i^y - x_j^y\right)^2}$. (See figure 4). This

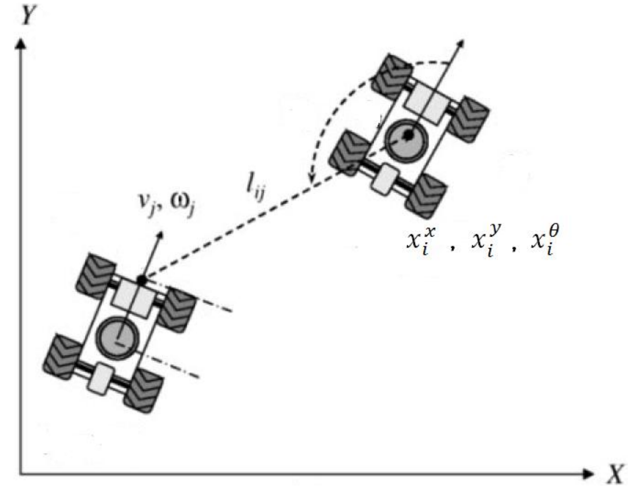

Fig. 4. Picture of two wheeled vehicles.

nonlinear model can be used in advanced control problems when analysis of actual dynamics of the vehicle is studied along with the cooperative control design. This will be used in other place in the report. When the cooperative algorithms are to be studied with the assumption of the well-done designed vehicles, a simpler dynamic model is considered as in next section.

#### 2) Linear Vehicle Dynamic Model

As the global problem of cooperative control is concerned with the interconnections between agents (vehicles) and the higher-level control strategies, nonlinear dynamic can be dropped. So, the vehicle can have the dynamic model of

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i \\ y_i &= Cx_i \end{aligned} \tag{4}$$

With $A \in \mathbb{R}^{m \times m}$ and $B \in \mathbb{R}^{m \times p}$. This model can give us the flexibility of solid study of the higher-level control of the group and the interconnections between the *N* vehicles. Also, the model will give us the chance of designing the proper decentralized control input $u$. One option of a model can treat the vehicle as the double-integrator model

$$\ddot{x}_i = u_i \tag{5}$$

Put in mind that all states interact with each other through the topological arrangement of the vehicles, i.e. communication graph. This communication graph is deployed according to the sensory information available. As mentioned before, with all vehicles having access to all others' states, nature of communication can be assumed to be from *wireless bidirectional channels*. Another kind of sensory information can be utilized by the technology found onboard of the vehicle, such as vision processing system with a camera or ultrasonic range sensors As in figure 1, nodes represent the vehicles and edges represent the 'interaction' between the vehicles. All details about graph description of certain group will be explained in next section.

### B. Graph Theory

The communication structure of the group of $N$ agents is described by a graph. *Graph Theory* gives us the complete mathematical correspondence to what we want. With [7] and [8] the main papers giving us the introduction, this preliminary information about graph theory with relation to our need is presented. One should inform the reader that here we are dealing with the *algebraic graph theory*.

- A *directed/undirected* graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ consists of a *vertex* set $\mathcal{V}$ and an *edge* set of $\mathcal{E}$.
- An *edge* is defined as any ordered pair of distinct vertices (for a directed graph, $(\alpha,\beta) \neq (\beta,\alpha)$ and $(\alpha,\beta) = (\beta,\alpha)$ for undirected graph).
- If $\alpha,\beta \in \mathcal{V}$, i.e. are vertices, and $(\alpha,\beta) \in \mathcal{E}$ , i.e. form an edge, then, $\alpha\ and\ \beta$ are said to be *adjacent*.
- A *path* from $\alpha$ to $\beta$ is the sequence of distinct vertices starting with $\alpha$ and ending with $\beta$ such that any two consecutive vertices are adjacent.
- Graph $\mathcal{G}$ is said to be *connected* if there exist a path between any two vertices in $\mathcal{G}$.

To relate graph theory with control theory, matrices is associated with the graphs. With vertices are associated with agents (vehicles), they, vertices, can be denoted as $\alpha_i$, giving us $N$ vertices.

- The *normalized adjacency matrix* of a graph $\mathcal{A}(\mathcal{G})$ is a square matrix of size $N$ indexed by the vertices.
- Element $\mathcal{A}_{ij} = 1/d(\alpha_i)$ if $(\alpha_i,\alpha_j)$ exists. $\mathcal{A}_{ij} = 0$ otherwise. $d(\alpha_i)$ is the number of edges *going out* of $\alpha_i$ .
- The normalized *Laplacian* of a directed graph is defined by

$$L = I - \mathcal{A} \tag{6}$$

Furthermore, the *neighborhood* $\mathcal{N}_i$ of a agent $x_i$ is defined by

$$\mathcal{N}_i = \{ j \in \mathcal{V} : \mathcal{A}_{ij} \neq 0 \} \tag{7}$$

This means that the collection of agents $j$ ‘seen’ by agent $i$ is the neighborhood of it. In other words, agent $i$ has the neighborhood of all agents that are adjacent to it.

- The *Laplacian* of a graph is defined as

$$L = [L_{ij}],\quad L_{ij} = \begin{cases} -1, & j \in \mathcal{N}_i \\ |\mathcal{N}_i|, & j = i \\ 0, & otherwise \end{cases} \tag{8}$$

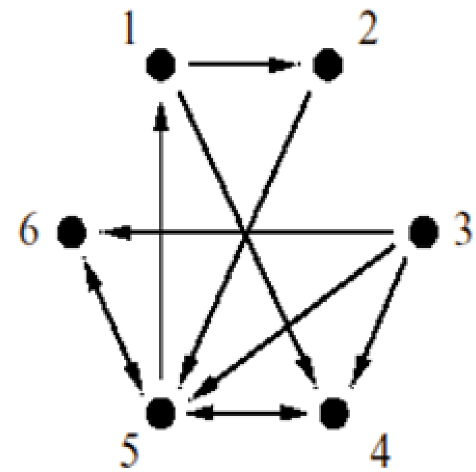


Fig. 5. A connectivity graph of 6 vehicles.

- $|\mathcal{N}_i|$ equal to the number of agents in the neighborhood of agent $i$
- the Laplacian of a graph defines the connectivity between vertices.
- The Laplacian is a *row stochastic matrix*, which means that the sum of each row equal zero.
- $L$ is a positive semi-definite matrix.
- $L$ is symmetric for an undirected graph.
- The multiplicity of zero eigenvalue of $L$ is equal to the number of connected edges.

The above definitions can be visualized in figure 5. In that figure, a connectivity graph is shown for 6 vehicles. Figure 5 shows a *directed* graph. The normalized Laplacian associated with the graph is

$$L = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 0 & 1 & -1 & 0 \\ -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & 1 & -\frac{1}{3} \\ 0 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 1 \end{bmatrix}$$

From figure 5 and associated Laplacian we can say that vehicle 1 can only ‘see’ vehicles 2 and 4. Vehicle 2 can ‘see’ only vehicle 4, and so on. Line of sight of a vehicle is defined by the sensory information of that vehicle only. Note that $L$ can be time-varying.

## V. Cooperative Control Models

You can consider this is the real part relating to cooperative control. In this section, detailed discussion of different models and strategies of cooperative control is presented. Before further analysis, the complete multi-vehicle system defined in (4) can be represented by

$$\dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{BU} \qquad (9)$$

With

$$\boldsymbol{X} = [x_1 \quad x_2 \quad \cdots \quad x_N]^{\boldsymbol{T}},$$
$$\boldsymbol{U} = [u_1 \quad u_2 \quad \cdots \quad u_N]^{\boldsymbol{T}}.$$

And

$$\boldsymbol{A} = I_N \otimes A,$$
$$\boldsymbol{B} = I_N \otimes B \qquad (10).$$

The operation $\otimes$ is called the *Kronecker Product*. This operation is defined by

$$Q \otimes P = \begin{bmatrix} q_{11}P & q_{12}P & \cdots & \cdots \\ q_{21}P & q_{22}P & & \\ \vdots & & \ddots & \\ \vdots & & & \ddots \end{bmatrix} = \left[q_{ij}P\right] \qquad (11)$$

One can see the new augmented system of the form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} A & 0 & \cdots & 0 \\ 0 & A & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} B & 0 & \cdots & 0 \\ 0 & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}$$

In (8), $\boldsymbol{X} \in \mathbb{R}^{(N \times m) \times 1}$, $\boldsymbol{U} \in \mathbb{R}^{(N \times p) \times 1}$, therefore, $\boldsymbol{A} \in \mathbb{R}^{(N \times m) \times (N \times m)}$, $\boldsymbol{B} \in \mathbb{R}^{(N \times m) \times (N \times p)}$. So, in (9), the whole group of vehicles are now characterized by $\boldsymbol{A}, \mathbf{B}$. This will let more freedom in studying the multi-vehicle system for further higher-level of control, namely cooperative control. You can see clearly that each agent is not in any way coupled with another one. We assume here also that any vehicle can have the values of others' states if they are from the neighborhood of the vehicle. In more simple illustration, vehicle *i* have access to all $x_j$ for all $\mathcal{N}_i$ defined in (7).

In order to have a solid analysis and design for a high-level control of the group, the interconnections between vehicles should be translated to a mathematical way. We define a *performance output* $z_i$ for vehicle *i* and defined by

$$z_i = G(x_i) = \sum_{\forall j \in \mathcal{N}_i} g\left(x_i - x_j\right) \qquad (12)$$

With $g\left(x_i - x_j\right)$ defines interaction between vehicle *i* and vehicle *j*. $g(.)$ can be called the *interaction function*. This function is related in some way to the connectivity graph of the vehicle under analysis. Having (9) and (12), the stage is set for further cooperative control study of methods and algorithms. First in section V-A, the early models of cooperative systems are studied to have a feeling about the original work in the field. Then in section V-B, an overview of different *Consensus* algorithms is presented. V-C will conclude with study of *Formation Control* strategies. Stability Analysis will also be discussed in all parts. Remember that *Control Design* will be delivered in Section VI. Sections V and VI have been made separate for giving an independent look upon cooperative control strategies analysis in one hand and the process of design in the other.

### A. Early Models

The models discussed here are coming from studies inspired by biological systems. The work of Vicsek *et al.* presented in [10] gives us a view of the swarm behavior of different species in nature. Without loss of generality, here we present a discrete-time model which has no different implications from the continuous one. Both reflect the same thing in terms of concept. We assume here that we are interested only in the headings of agents; here are birds, fish, ants, etc. So, a coordinated motion of the group is governed by

$$x_i^\theta(k+1) = \frac{1}{1 + |\mathcal{N}_i(k)|}\left(x_i^\theta(k) + \sum_{\forall j \in \mathcal{N}_i(k)} x_j^\theta(k)\right) \qquad (13)$$

With $|\mathcal{N}_i(k)|$ equal to the number of agents in the neighborhood of agent *i* at time instant *k*. Eq. (13) tells us that next heading update is in some kind of a weighted average of the preceding headings available for an agent. The model in (13) started the boom of research in this field. Model defined by (13) is called Vicsek's Model.

According to [10], stability is maintained for the whole group. Stability in a cooperative system is reflected by the *convergence* to the desired configuration as $t \to \infty$. we here are assuming that the neighborhood of agent *i* is not fixed. This also means that the connectivity graph $\mathcal{G}$ is time-varying. Actually the connectivity graph $\mathcal{G}$ is changing a *switching* fashion. So for the group to be stable, i.e. converge to some $\hat{x}_i^\theta$ for all *i*, then the union of all graphs $\mathcal{G}(k), \mathcal{G}(k+1), \ldots, \mathcal{G}(\infty)$ should form a *connected* graph (refer to definition of a connected graph in IV-B).

### B. Consensus Algorithms

Here in this section, information about Consensus methods is discussed. Many problems in cooperative control can be considered as a consensus problem, as explained earlier. We will show here different kinds of algorithms. We mean by consensus that

$$\left\|x_i - x_j\right\| \to 0 \;\; as \;\; t \to \infty \qquad (14)$$

Notice that (14) is equivalent to (1). To fulfill the consensus rule in (14), one can suggest a behavior for vehicle *i* by

$$\dot{x}_i = -\frac{1}{|\mathcal{N}_i|}\sum_{\forall j\in\mathcal{N}_i}(x_i - x_j) \tag{15}$$

In (15), vehicle's state is updated as an average of information available for it. For more generalization of consensus problem, one can define a consensus 'protocol, by

$$\dot{x}_i = \sum_{\forall j\in\mathcal{N}_i} a_{ij}\cdot(x_i - x_j) \tag{16}$$

Eq. (16) defined a general rule for consensus of a group of vehicles. with $a_{ij}$ being a relative weight between vehicles *i* and *j*. If vehicles *i* and *j* communicate in a bidirectional way, then one can assign $a_{ij} = a_{ji}$. One can observe that the group of vehicles that have bidirectional interconnections, have a undirected connectivity graph. We can say that for the group of vehicles defined by the dynamics in (9) that have a connectivity graph $\mathcal{G}$, a desired collective dynamics of the group can be defined by

$$\dot{\boldsymbol{X}} = -L\boldsymbol{X} \tag{17}.$$

With $L$ being that Laplacian of the associated with $\mathcal{G}$. You can see that for bidirectional communication between vehicles lead to a symmetric $L$. When the graph associated with the group found to be undirected, it has been found that the states converge, i.e. stabilize, if $\mathcal{G}$ is *connected*. You can check the eigenvalues of the Laplacian. It is found also that the convergence point is the average of the initial state values of the vehicles. When the graph associated is directed, this does not imply that the group will converge.

Furthermore, generally, for a connected graph, stability is maintained; and for unconnected one, the system becomes unstable. However, if the communication between vehicles is switching arbitrary ($L$ is time-varying), due to different issues that the reader can imagine why, stability is not reached unless a condition is fulfilled. From [10], this condition can be summarized into:

"*Consensus is reached asymptotically if there exist infinitely many consecutive bounded time intervals such that the union of the graphs over such intervals is totally connected*".

Note that in the above condition, 'time intervals' is defined. That is because that as we are analyzing a continuous-time model of the system, switching graph is triggered in discrete manner. Note that usually, $L(t)$, i.e. time-varying Laplacian, can not be defined explicitly.

As the consensus problem is the mother of all other cooperative control problems, discussion done above will benefit further study for different strategies.

## C. *Formation Control*

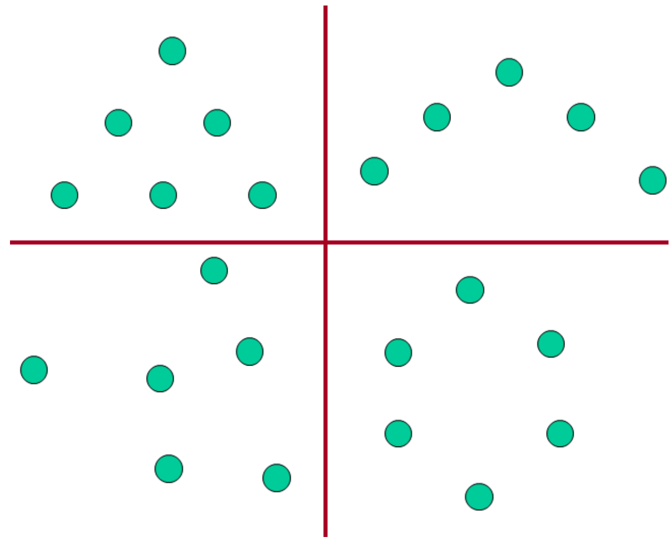

Fig. 6. An example of four different formations. *Nodes* representing the vehicles.

Formation control is not, by any means, different than consensus problem. However, because of the great importance of designing dynamic vehicle formations, a branch of research stands alone to study different kind of problems. In formation control, the goal is not only to agree on a common position. It requires that the group of vehicles maintain predefined relative positions between one another. Formations are defined in the state space of the agents. For multi-vehicle systems, formations are defined in 2D space. Examples of different formations are shown in figure 6.

The most simple consensus algorithm representing a formation control problem is given by

$$\dot{x}_i = \sum_{\forall j\in\mathcal{N}_i}(x_i - x_j - r_{ij}) \tag{18}$$

In (18), $r_{ij}$ defines the preconfigured distance (reference) between vehicle *i* and vehicle *j*. So, analyzing (18) gives us that the group of vehicles will converge to their defined inter-distances. Note that when $r_{ij} = 0$, it becomes the normal consensus problem. Here in this section of the report, talk about formation control will only introduce the concept with accompanied analysis of the group system under formation study. Control design will be discussed in section VI. Along with keeping distance between vehicles controlled, one other objective is to move the group as a whole following a trajectory while maintaining its formation. This problem is also can be treated as a formation control problem.

**Artificial Potential Fields**. One big area of research that is related to vehicles formations in general is the area of *Artificial Potential Fields*. With many applications elsewhere, it has great importance in defining formations of vehicles. One other application of artificial potential fields is also related to mobile vehicles, but not related to cooperative control, is *Obstacle Avoidance*. It is 'artificial' because we, designers, define them for our own interest. They do not exist by nature. A potential field force around vehicle $i$ can be defined like the

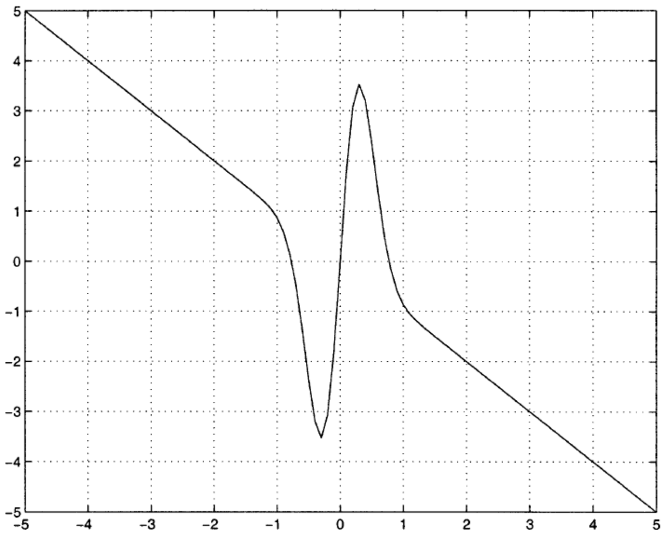


Fig. 7. Force vs. distance. It employs both repulsion and attraction effect depending on how distant is the vehicle to its neighbor.

attraction function in (12). One can consider the example depicted in [3]. For a swarm, rather than representing desired distances like (18), an attraction/repulsion function is defined by

$$\dot{x}_i = \sum_{\forall j \in \mathcal{N}_i} g(x_i - x_j) \quad (19)$$

$$g(x_i - x_j) = -(x_i - x_j)\left(a - b \cdot exp\left(-\frac{\|(x_i - x_j)\|^2}{c}\right)\right) \quad (20)$$

Eq. (20) define an artificial attraction/repulsion mechanical-like force around vehicle $i$. To visualize this artificial *force*, figure 7 show the value of force $g(x_i - x_j)$ versus distance $(x_i - x_j)$. In Fig.7, (20) is evaluated in one-dimensional space of $x$ with $a = 1, b = 20, and\ c = 0.2$. 2D of even 3D forces can be imagined. $g(.)$ force is repulsive when vehicles are too close and attractive when far. Also a defined force should be selected so that it diminishes for too far distances. If strategy of (20) is used, vehicles will converge to common region with average position $\bar{x} = (1/|\mathcal{N}_i|) \sum_{\forall j \in \mathcal{N}_i} x_j$ . The value $\bar{x}$ makes the center of the swarm. All vehicles will have distances from the center with $\|(x_j -$ *x≤(bac2*exp*−12)*. Here, (20) is just an example. More complicated and sophisticated functions can be studied.

Above discussion of artificial potentials will help to understand how formation control could be used. In a formation control problem, the desired formation can be formulated also as a directed graph $\mathcal{G}_d$. However, This graph is characterized by 1) a set of desired edges $\mathcal{E}_d$, 2) the same set of vertices of the group of vehicles $\mathcal{V}$, and 3) the added elements $\{d_{ij}\}$ of the desired relative distances for all $i$ and $j$ forming edges $(i,j) \in \mathcal{E}_d$. One can visualize the desired formation by the *weighted* graph $\mathcal{G}_d$ with edge weights being related to $d_{ij}$ . From [5], so a *local* behavior for a vehicle $i$ can be defined by

$$\dot{x}_i = -\sum_{\forall j \in \mathcal{N}_i, \forall (i,j) \in \mathcal{E}_d} g(x_i - x_j - d_{ij}) \cdot (x_i - x_j - d_{ij}) \quad (21)$$

You can see from (21) that the motion of vehicle $i$ depends on the attraction/repulsion force value and the distance error. Note that summation is computed with the aid of all information available, i.e. desired edges and neighborhood of $i$. Different artificial forces used lead to different behaviors.

Other tasks of formation control can be added. One is to follow a trajectory while maintaining formation. Another task can be done for group splitting and rejoining. Shape reconfiguration due to obstacles or a higher-level command. As formation control is a consensus problem, connectivity graph of the group of vehicles should have the same conditions of convergence (stability) along with conditions for formation convergence.

## VI. Cooperative Control Design

This section reserved for the process of designing a decentralized control in order to fulfill the different objectives. Objectives considered can be formation control or consensus algorithm. In previous section, different kinds of cooperative control models are discussed with thorough stability and convergence analysis.

As the goal is to design identical decentralized controllers in each vehicle we have, these local controllers should keep high-level objectives met. So a complete view of the problem can be visualized in figure 8. We remind the reader that vehicle collective dynamics is defined in (9). The problem has no coupling between vehicles. Only the coupling is coming from the cooperation in the *feedback* via the shared connectivity graph. Before any further analysis, assumptions should be mentioned:

- Local dynamics of the vehicle (inner loops) is stable.
- Vehicles have access to other *seen* vehicles' states. So we can assume a state feedback control.

In order to have all the shared communication information, we set the overall data available utilizing (12) by defining

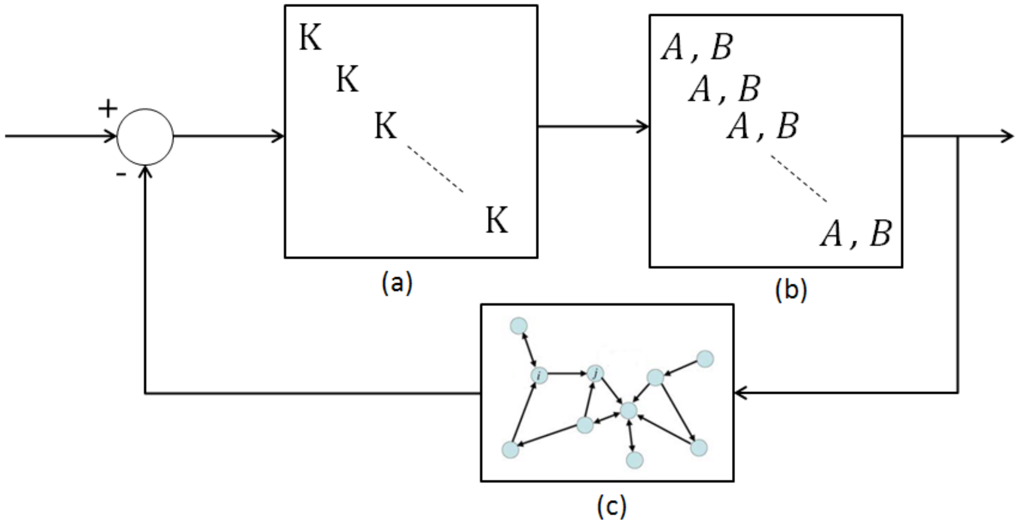


Fig. 8. Complete picture of the control problem at hand. (a) the decentralized controller gains. (b) Plant (vehicles) dynamics. (c) Sensory information in a network

$$\boldsymbol{Z} = \boldsymbol{G}(\boldsymbol{X}) \quad (22).$$

With $\boldsymbol{Z} = [z_1{}^T \quad z_2{}^T \quad \cdots \quad z_N{}^T]$ being group performance output associated with all $N$ vehicles. $\boldsymbol{G}(.)$ is the group complete interaction function as all-vehicle form of (12). Usually, this interaction function is representing the connectivity graph of the group.

One can think that a typical decentralized control input should be designed as

$$u_i = f(z_i, x_i) \quad (23).$$

In (23), for vehicle $i$, the control action will depend on the current state of it and on the information available in its neighborhood. Note here that $f(.)$ is unified for all vehicles. So, the control design should be done properly so that can accommodate different scenarios. So a full system control input can be

$$\begin{aligned} \boldsymbol{U} &= \boldsymbol{F}(\boldsymbol{Z}, \boldsymbol{X}) \\ &= \boldsymbol{F}(\boldsymbol{G}(\boldsymbol{X}), \boldsymbol{X}) \end{aligned} \quad (24).$$

The closed-loop system that governs the vehicles can be seen as

$$\dot{\boldsymbol{X}} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{B}\boldsymbol{F}(\boldsymbol{Z}, \boldsymbol{X}) \quad (25).$$

In this section of the report, we will start by introducing control input designs for general consensus problems in VI-A. Then, more sophisticated control design for formation control is presented in VI-B. Section VI-C will give some short solution for other cooperative control problems.

### A. Consensus Control Design

Consensus convergence has been discussed in detail previously. Here, further analysis along with design criteria is presented. Before further discussion, the dynamics of the group of vehicles that we want to control is defined. We can use (2) to generally describe the vehicle dynamics. We can assume, for simplicity, the $N$ vehicles each having the identical dynamics described by (4). We can assume also that each vehicle is governed by a single-integrator dynamics. This assumption will give us a special case of (4). We assume that the output of the vehicle system is the state of it. You should note that for the performance output defined in (12) for a vehicle $i$, the neighborhood of this vehicle should not be empty. So by this we can suggest a decentralized control input to be

$$u_i = f(z_i, x_i) = -Ke_i \quad (26).$$

With $K \in \mathbb{R}^{p \times m}$ and $e_i$ to be some local *error* evaluation. So, the problem is summed up to the point of the proper choice of an error that relate to all information available for a vehicle, i.e. shared information through the communication network. One straight forward selection of the error is the performance output. So a suggestion is to have

$$e_i = z_i = \sum_{\forall j \in \mathcal{N}_i} g\left(x_i - x_j\right) \quad (27)$$

However a selection of an interaction function should be adopted. An obvious choice is to have it as the weighted average found in (16). So, in the same notion, a proposed decentralized control could be

$$u_i = -K \sum_{\forall j \in \mathcal{N}_i} a_{ij} \cdot \left(x_i - x_j\right) \quad (28)$$

You can see here that the control input utilizes all available information, i.e. information around the neighborhood. As the same analysis of (16), letting $a_{ij}$ be weights coming from the connectivity graph $\mathcal{G}$ between the group of vehicles, then by some mathematical manipulation, the all-vehicle system control becomes

$$\boldsymbol{U} = -(I_N \otimes K)(L \otimes I_m)\boldsymbol{X} \quad (29).$$

With, as mentioned previously, *m* is number of internal states of each vehicle, $L$ is the Laplacian associated with connectivity graph $\mathcal{G}$; and the operation '$\otimes$' defined in (11). You can see from (29) that it shows a state-feedback algorithm; and with feedback $(L \otimes I_m)\boldsymbol{X}$ which is called the *consensus feedback*. So the group closed-loop system could be shown by, from (24), (25) and (29),

$$\dot{\boldsymbol{X}} = \boldsymbol{AX} - \boldsymbol{BK}(L \otimes I_m)\boldsymbol{X} \quad (30).$$

With the group control gain matrix $\boldsymbol{K} \in \mathbb{R}^{(\boldsymbol{N}\times\boldsymbol{p})\times(\boldsymbol{N}\times\boldsymbol{m})}$, is defined by

$$\boldsymbol{K} = (I_N \otimes K) = \begin{bmatrix} K & 0 & \cdots & 0 \\ 0 & K & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K \end{bmatrix} \quad (31).$$

Remember that for vehicle *i*, its control input is defined only by feeding back available information in the neighborhood of the vehicle, i.e. the control is decentralized. You can see this in (28). Here we only employed a constant gain control. However, you can design other decentralized controller systems to have other effects, e.g. integral action. You can see more in [1].

### B. Formation Control Design

We saw in previous section the utilization of consensus feedback to help of convergence of the vehicles. As formation control problem is considered to be a consensus problem also, quite similar approaches could be made. So, by the same notion of (18), a control solution can be chosen as

$$u_i = -\sum_{\forall j \in \mathcal{N}_i} a_{ij} \cdot (x_i - x_j - d_{ij}) \quad (32)$$

With $d_{ij}$ being the inter-distances between vehicles. Looking for a graph-based decentralized control design, some modification can be made. In [5], a proposal for general decentralized control is the same as (28) but with the definition of $a_{ij}$

$$a_{ij} = g(x_i - x_j - d_{ij}) \;\; , \;\; \forall (i,j) \in \mathcal{E}_d \quad (33)$$

With $\mathcal{E}_d$ defined in V-C as the set of edges of the desired connectivity graph $\mathcal{G}_d$ . This graph is also characterized also by $d_{ij}$ as edges weight. So the proposed decentralized control can be defined as

$$u_i = -\sum_{\forall j \in \mathcal{N}_i, \forall (i,j) \in \mathcal{E}_d} g(x_i - x_j - d_{ij}) \cdot (x_i - x_j - d_{ij}) \quad (34)$$

The weights $g(x_i - x_j - d_{ij})$ can be inserted to the *consensus feedback* defined in VI-A to make modified form of the Laplacian. This modification is done by making a weighted Laplacian. This weighting can be defined as

$$L_w = (L^{1/2})W(L^{1/2})^T \quad (35)$$

With $W$ is a diagonal matrix. Diagonal elements are assigned as the weights on edges of the desired graph. We can say that

$$W = diag(g(x_i - x_j - d_{ij})) \quad (36)$$

For all $(i,j) \in \mathcal{E}_d$, i.e. we can see that number of diagonal elements equal to number edges defined in $\mathcal{E}_d$. As in [5], $L^{1/2}$ is called the *incidence matrix*.

We can see that from (34) the importance of designing the weights. Also, control designed in (34) can work even for dynamically changing connectivity graph such that convergence conditions (see section V-B) are met. You can see that the weights will often be evaluated according to some potential field, discussed in V-C.

Other approaches related to formation control are the concepts of 'leader-following' and 'virtual-leaders'. In the introduction of a group leader or more, a decentralized control solution can be defined as

$$u_i = -\sum_{\forall j \in \mathcal{N}_i} g_1(x_i - x_j - d_{ij}) - \sum_{\forall j \in \mathcal{L}} g_2(x_i - x_j) \quad (35)$$

With $\mathcal{L}$ is the set of leader vehicles; and $g_2(.)$ is an interaction function defined for behavior with leaders.

Other approach is to put the formation control problem as an *optimization problem* with the goal of minimizing of a cost that could be defined as being a formation error. Other physical constraints could be added to the optimization problem.

### C. Other Cooperative Control Designs

Here is just a brief a section for other problems under cooperative control. We mainly here give the idea behind *Flocking* control and *Rendezvous* problem. For flocking control, the three rules of Reynolds (see II-B) should be fulfilled. So a logical decentralized control solution should be constructed as

$$u_i = f_1 + f_2 + f_3 \qquad (36).$$

With $f_1$ could be a potential field function; $f_2$ could be a damping term to regulate velocities with the neighborhood; and $f_3$ is responsible for the global group objective. In a rendezvous problem, all vehicles are required to reach a specific position simultaneously, i.e. in the same time. So, a formulation for this problem suggested in [13] is to define a rendezvous region around the desired rendezvous point with radius $\delta$ . we define a quantity $\rho$ defined by

$$\rho = \frac{\max(\|x_i(t_\rho)\|}{\rho}$$

With $t_\rho$ is the time of arrival of the first vehicle in the region. So, the problem is solve by designing a decentralized control such that a desired $\rho_d$ be $\rho \le \rho_d \le 1$. A perfect rendezvous is when $\rho = 1$.

## VII. Research Directions in Cooperative Control

Here in this brief section, some enlightenment will be given about different research directions in cooperative control. So, until now, the discussion of cooperative control has been in detail. But some other investigation could be done in other aspects related to the main problem.

**State Estimation**. Throughout the paper, we assumed that whenever the connectivity edge is established, the vehicle will have the access to other vehicles states. However, in real-life problem, the case is often different. As mobile robot technology get more and more sophisticated, internal behavior and dynamics of a vehicle increases. So, it is logical to expect that accessibility to other vehicles state is by no means trivial.

Other issues could be addressed because the existence of noise in the environment. Sensors, actuators and communication devices will be interfered by different kinds of noises. So, accuracy of states is questioned. A solution to all of these problems is to have some kind of *decentralized state-estimators*. These estimators will be responsible for evaluating the state that will be provided to the decentralized controllers. Actually, state estimation problem in cooperative systems would have many interesting features.

The estimation goal in a cooperative environment, like our multi-vehicle system, is just to have an estimate of the relative states $(x_i - x_j)$ rather than the absolute ones $x_j$ in the neighborhood. This estimation could be designed for example to utilize some *vision processing* techniques to estimate the inter-distances between vehicles. Other sensors could be utilized. *Cooperative Estimation* is a big branch of research that's stands alone.

**Hybrid Control**. The area of hybrid control is booming in recent years. The mixture of *Control Theory* and *Computation Science* proved to have great solution to many discrete-by-nature dynamic systems, i.e. system with discontinuities in its definitions (non-smooth dynamics).

Switching (discontinuous) phenomena is expected when we have discrete elements in the cooperative control problems. These elements could be: the non-fixed number of agents (scalability) in the system, which makes a non-smooth dynamics of the group; another issue is the piecewise changing of the group objectives; and also the discontinuities in the communication environment. Other issues can be imagined.

**Stochastic Approaches**. The nature of randomness is expected in big systems such cooperative systems. Here we are not discussing only the issue of noise. We here specifically discuss the switching changing of connectivity graphs. We assumed previously that connectivity graph is time-varying. However, usually, communication network between vehicles is not governed by a known function. So, a stochastic analysis can be done to investigate hoe every element in the graph is

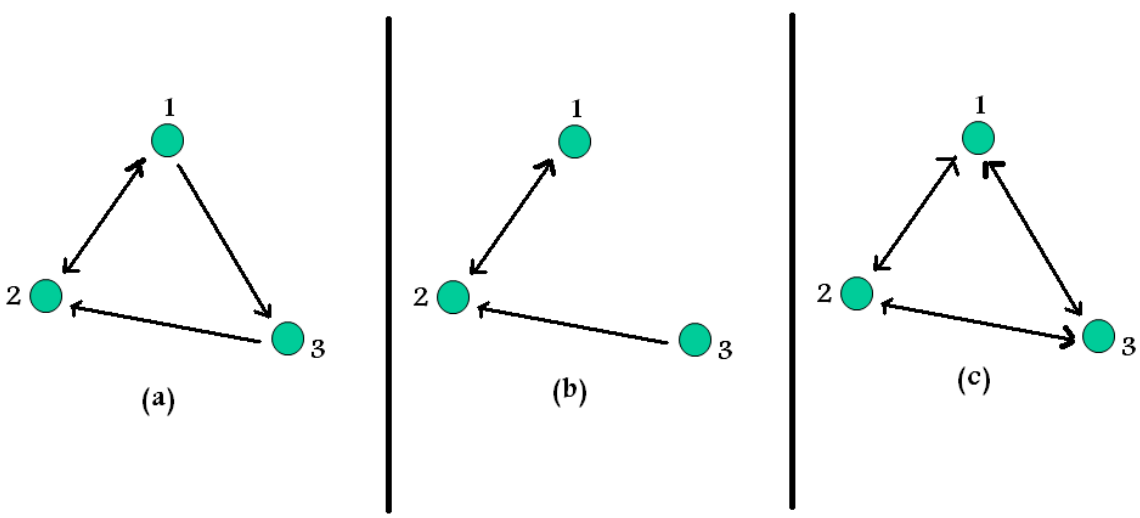


Fig. 9. Three different connectivity graphs. (a) normal kind of interconnection. (b) there is no connection between vehicles 1 & 3. (c) complete connected graph with vehicles seeing each other.

changing. As been discussed earlier, the condition of total connectivity of the changing graph mentioned previously should be ensured. If the graph 'jumping' structure can be modeled as a stochastic process, convergence analysis will be easier.

**Time-delay Systems**. Another field of research is that when we have delays in information exchange between vehicles. This problem can not, by any means, to be considered trivial. The loss of synchronization will lead to many problems specially in executing the decentralized control.

## VIII. SIMULATION EXAMPLES

Here in this section, some simulation experiment is done to illustrate more about the cooperative control area. Here, we are doing simulations in MATLAB environment. We assume here a 3-vehicle labeled as $i = \{1,2,3\}$. So, here, we have $N$ = 3. The vehicles move in a 2D plan with identical first-order dynamics. We assume having two input, $p$ = 2, $u_i = \begin{bmatrix} u_i^x & u_i^y \end{bmatrix}^T$. Also, the states of each vehicle are defined by $x_i = \begin{bmatrix} x_i^x & x_i^y \end{bmatrix}^T$ with $m$ = 2. Being the x- and y- coordinates in two-dimensional space. So, a complete state space representation of a vehicle is

$$\dot{x}_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_i + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u_i \qquad (37).$$

Eq. (37) is equivalent to $\dot{x}_i^x = u_i^x$ and $\dot{x}_i^y = u_i^y$. You can picture these vehicles with *two* independent throttles acting on each axis direction with *friction* on the motion (the first-order dynamics). We try different connectivity graphs shown in figure 9, with nodes being the vehicles and directed edge being interconnection between the three vehicles. For example, in figure 9(a), vehicle 1 can see both vehicle 2 and 3. Vehicle 2 sees only vehicle 3; and vehicle 3 sees vehicle 1. The associated Laplacian with the graph shown in fig. 9(a) is

$$L = \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

So the expected behavior of the system should be defined as

$$\dot{\boldsymbol{X}} = -(L \otimes I_m)\boldsymbol{X} \qquad (37).$$

With $\boldsymbol{X} = \begin{bmatrix} x_1^x & x_1^y & x_2^x & x_2^y & x_3^x & x_3^y \end{bmatrix}^T$. Let the initial positions of the vehicles to be $\boldsymbol{X}(0) = [5 \quad 7 \quad 1 \quad 3 \quad 10 \quad 1]^T$. This means that in a 2D plan with a defined by origin (0,0), vehicle 1 will initially be at (5,7). It should be expected, if a stable connection, that all vehicles go to the average position among all, i.e. $x_i \to \frac{1}{3}\sum x_j(0)$. Simulation is done for all three different graphs. The results are shown in figure 10. Every row of plots show reflect graphs (a), (b) and (c) respectively as defined in figure 9. First column of plots reflects the motion on x-axis; and second column reflect motion on y-axis. Each plot shows the 3 vehicles.

You can see that for graphs in figure 9(a,b), it turned out that vehicles converged to their average positions. Even that in graph (b) vehicle 1 and 3 do not see each other, but convergence occurred because of that the graph is connected, i.e. there is a path for information flow between all vehicles. Slight difference in speed in graphs (a) and (b) is observed. The most unexpected result is that even when the vehicles are well connected with each other (complete connection), vehicles diverge. Vehicles do reach their average positions. This is due to the values of eigenvalues of the Laplacian matrix. This problem has been analyzed and justified in [1].

## IX. CONCLUDING REMARKS

At the end of this long report/paper, cooperative control was shown to be an emerging field in research. Cooperative control has the nature of multidisciplinary aspect. It involves many tools towards the goal of coordinated tasks. It was shown that there exist decentralized solutions that can be employed in local systems and giving results matching global objectives. The powerful utilization of Graph Theory gave solid mathematical foundation to model communication interconnections. Many techniques under the field of cooperative control have interesting findings. The main principle of Consensus was set as the standard problem that included most of cooperation problems. Special tasks as formation control was discussed with detail in relation with other aspects in control theory. Applications and research status were discussed to give many possible areas of practical commercial use. Simulation was done to give a 'feeling' about multi-vehicle systems cooperating together with interesting results occurred.

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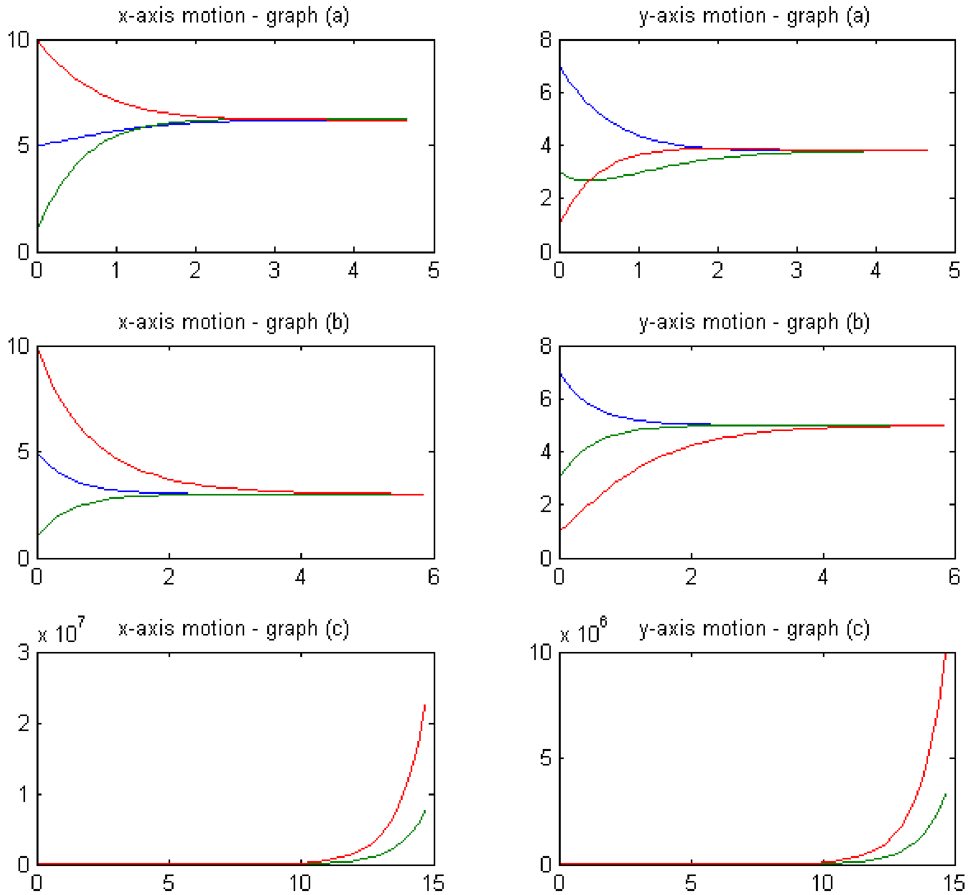


Fig. 10. Simulation Results.. Rows: results of each graph (a), (b) and (c). Columns: results on motion x- and y- axes.


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## APPENDIX

Here is just the MATLAB code used to generate the simulation.

```
N=3; %number of vehicles
L1=[1 -.5 -.5;0 1 -1;-1 0 1]; %graph (a)
L2=[1 -1 0;-1 1 0;0 -1 1]; %graph (b)
L3=[1 -.5 -.5;-.5 1 -.5;-.5 -1 -.5]; %graph (c)
X0=[5;7;1;3;10;1]; %Initial Positions

LM1=kron(L1,eye(size(A)));
sys1=ss(-LM1,zeros(6,1),eye(6),zeros(6,1));
LM2=kron(L2,eye(size(A)));
sys2=ss(-LM2,zeros(6,1),eye(6),zeros(6,1));
LM3=kron(L3,eye(size(A)));
sys3=ss(-LM3,zeros(6,1),eye(6),zeros(6,1));

[y1,t1,x1]=initial(sys1,X0);
[y2,t2,x2]=initial(sys2,X0);
[y3,t3,x3]=initial(sys3,X0);

subplot(321);plot(t1,y1(:,1),t1,y1(:,3),t1,y1(:,5))
title('x-axis motion - graph (a)')
subplot(322);plot(t1,y1(:,2),t1,y1(:,4),t1,y1(:,6))
title('y-axis motion - graph (a)')
subplot(323);plot(t2,y2(:,1),t2,y2(:,3),t2,y2(:,5))
title('x-axis motion - graph (b)')
subplot(324);plot(t2,y2(:,2),t2,y2(:,4),t2,y2(:,6))
title('y-axis motion - graph (b)')
subplot(325);plot(t3,y3(:,1),t3,y3(:,3),t3,y3(:,5))
title('x-axis motion - graph (c)')
subplot(326);plot(t3,y3(:,2),t3,y3(:,4),t3,y3(:,6))
title('y-axis motion - graph (c)')
```