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Energy-Efficient Motion Control of Mobile Robots

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I. INTRODUCTION

With great advancements in applications of Mobile Robots, concerns of energy grew. Most of (not all) research and work done on the field of robotics is developed with no relation to 'life' of the robot. Specifically, mobile robots use batteries to power themselves. So, the 'lifespan' of these robots is limited. So, Investigations on energy-related concepts are also of great importance. Motion control and planning of robot have, of course, relation to energy consumption. This report will shed the light on the concept Energy Efficiency in Mobile Robots. Also, a somehow detailed review of related work in literature is also shown. Focused study will also be done for specific work of Mei [2].

The report will start by introducing the concept of energy efficiency in general with relation to mobile robots in section II. Then, section III will discuss different classifications of energy consumption in mobile robots related to motion control and motion planning. The literature review of related work is then given in IV. Section V will discuss and analyze the work of Mei. Simulations and results analysis is in VI. Section VII, will end the report with giving future directions and conclusion.

II. ENERGY EFFICIENCY IN MOBILE ROBOTS

Generally, in evaluating machine performance, the question is raised to ask about how efficient that machine is. Efficiency is a measure to show, roughly, how much input is utilized to produce the output. We can say that efficiency is formulated (roughly) as in (1).

$$Efficiency = \frac{Output}{Input} \tag{1}$$

So, in the process of designing the machine and designing the operation of the machine, we want to have the most utilization of input to output. In other word, the goal is to have Efficiency maximized. So, better performance is attained by increasing the ratio in (1). In this section, first a brief familiarization of Mobile Robots is given in II-A. In part II-B, the concept of Energy Efficiency will be projected to Mobile Robots.

A) Mobile Robots

Mobile Robots can be defined as robots that have the option of movement in their environment. Unlike robotic manipulators, mobile robots have the ability to move their full 'body' across the environment and interact and react accordingly. Examples of Mobile Robot can be put as

- Wheeled robots, Car-Like Robots (CLR), Unmanned Ground Vehicles (UGVs). These are examples of mobile robots that operate on ground. A picture of one is shown in Figure 1.
- Unmanned Aerial Vehicles (UAVs) are mobile robots that fly rather than operating on ground.
- Another more complex example is Autonomous Underwater Vehicles (AUVs) which operate underwater.

Figure 1: a Wheeled Mobile Robot

Applications of mobile robots range from simply performing dull operations of automated cleaning to applications of combat or surveillance in warzones. So, many applications can be imagined that a mobile robot would perform fulfilling the objectives better than other methods.

B) Energy Efficiency in Mobile Robots

Mobility feature require independent power source rather than fixed sources. So in most of cases, batteries are used to power the robots. Other sources can be fuel, for example, for powering autonomous cars. But generally, batteries are the main sources of energy powering the robots.

It is obvious that batteries have finite limit. Energy stored in a battery is depleted with rate related to the consumption of the device equipped with. So, energy limit should be taken care of in designing the motion of the robot. Picture following applications and situations:

- Disasters: robots are distributed to find survivors and maybe also rescue them.
- Warzones: unmanned mobile robots are deployed to combat the enemies.
- Social: robots could be responsible of cleaning the floor or assisting people.
- Also imagine the cost of replacing or recharging the battery.

Above situations shows typical scenarios of operations of mobile robots. In critical situations of war of disaster, energy management should be optimal in order to elongate lifespan of the battery. Cost of replacing or recharging batteries also show us that proper energy utilization is of great importance. Cost minimization is crucial in designing robot operation. So, from above explanations, we can see that a main goal of design is to escape from going out of power. To have clear view of energy efficiency in mobile robots, we can make a definition in (2).

$$Efficiency = \frac{Output\ Task}{Energy\ Consumption} \quad (2)$$

'Output Task' can be the distance travelled by the robot, operation time, coverage area of the robots. In other words, these are the objectives put by the developers and designers. 'Energy Consumption' obviously means how the battery power is utilized to power different components of the robot. So, to increase the efficiency of our machine, i.e. the robot, we have to get the maximum tasks accomplishments and in the same time minimum consumption of energy. So, the ultimate goal of energy-efficient motion control of mobile robots is to *minimize energy consumption*. In next section, discussion about energy consumption in mobile robots is exposed.

III. Energy Consumption in Mobile Robots

Here, in this section, relatively detailed discussion is employed to reveal different sources of energy consumption in a typical mobile robot. From literature and experience, a rough classification of how energy is consumed is related to either:

1- Robot Actuators (Motors): III-A
2- Motion Planning: III-B
3- Motors & Planning integrated: III-C
4- Auxiliary Sources: III-D

In this section, the above four situations are discussed.

A) Motors Energy Consumption

Here, we will discuss how robot actuators, mostly motors, are consuming energy. Mobile robots in small-scale usually use DC motors as actuators acting on the environment. As, generally, mobile robots move in their environment via the equipped wheels. Rotation of the wheels is controlled by DC motors. So, motion is directly depending on the DC motors. A typical DC motor is current or voltage supplied depending on the circuitry accompanying the rotary body. So battery consumption is related to 'consumption' of the electrical signal required. A DC motor being a physical system, energy loss appears due to, say, friction or load Inertia or simply due to power consumption of the electronic circuit driving the motor. Figure 2 shows a schematic of Brushed DC Motor.

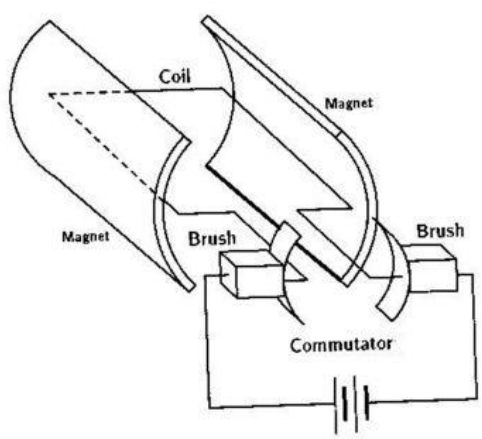


Figure 2: Schematic of Typical DC Motor

A solution of "Energy Loss" in a motor is simply to design a better motor with more efficient design to compensate for losses due to, say, friction. However, here we discuss only how to control the motion to minimize consumption irrespective of what actuators used. So, our goal here is to minimize the dissipated (loss) energy by minimizing the total input power supply to the motor.

Power consumption is found to be proportional to the velocity of rotation of the motor. So in order to manage the energy, each motor has to have a Power Model that captures the relation between motor variables, i.e. velocity or acceleration, and input power. To model this behavior, each problem should be treated as separate because of the dependence on motor structure. So, for some motors, power consumption increases linearly with velocity. In the other hand, others will behave exponentially for high velocities. The power model of a motor depends on motor design and electronic circuit driving the motor.

To solve the problem of energy loss in motors, motor controller should be designed optimally. This problem is called Loss-Minimization. So, rotation (and velocity and acceleration) of the motor would be generated according to optimal control problem with cost function that includes the power model of the system (motor). Further details of different strategies are discussed in section IV.

B) Motion Planning & Energy

Here in this section, we will consider the case where we focus on the effect of motion planning to energy consumption. So, here, the study is done independent of motor energy losses. So, to minimize energy consumption, we should design 'optimal' paths for the robot. Optimality here can be related to many things that would affect energy consumption.

In most cases, optimality criteria are related to the amount of time spent in task or distance covered during the task. So, the goal would be to minimize the distance and/or time during the operation. So, the algorithm will generate optimal paths of minimum distance and/or time. This idea is coming from thinking of that the faster or shorter you reach destination, the least you spend energy.

One step higher in planning putting in mind the energy is to include in the cost function of the optimization problem evaluation of the power of control input along with cost of distance or time. Generally in most optimal control formulation, a typical objective function can be of the form in (3).

$$J = \int [x^T Q x + u^T R u + 1] \, dt \tag{3}$$

The above cost function in (3) is typically used to construct optimal control for linear system. As can be seen, cost consists of quadratic evaluation on states x(in our case: distance, velocity, etc), control input u and time span appearing by '1'. From above, you can see that the input power is just evaluated as the square of the input. For single control this could be put as u^2 Generally, time-optimal solution will involve bang-bang control.

C) Motor & Motion Planning: Integrated

Here in this section, energy-optimal solutions would consider both energy directly from motors and energy as consequence of motion planning. Unlike previous situation which only considered path, here analysis of energy is done as how different paths would affect motors energy consumption.

Imagine the following solutions:

- Distance covered is minimized. However, minimum distance could include multiple sharp turns in path. These turns will make the robot decelerate and accelerate as required by the path. Recall that energy consumption of motors is proportional to velocity. So, for such case, the solution would be optimal only in distance but not energy obviously.

- Time is minimized. A logical solution to minimize the time is to operate the robot in the fastest velocity. However again, faster the velocity, higher the energy consumption.

From above possible scenarios, we can deduce that planning the motion not considering energy consumption by motors is not efficient planning in the energy sense. So, a better solution would be to integrate both path planning and motor control to come up with optimal motion, energy-optimal that is. So then optimization criteria should include what ever objective needed along with the minimizing energy consumption which is modeled according to the actual power model of the robot. Further details of techniques will be given in next section IV. One problem that could arise from such 'optimal' solution is to have conflicting objectives. A common example is the conflict between energy and time. Generally, minimizing the time would increase energy consumption and vise versa.

D) Auxiliary Sources of Energy Consumption

Here in this small part, just a review of other sources of energy consumption will be given. These sources are either minor or unrelated to motion of the robot. In any mobile robot, components exists other that the actuators. Generally two kinds of components that consume energy also are the sensing elements and robot's brain, the microcontrollers and/or the onboard computer.

- Sensors: the sensing elements are essential parts of any robot. Sensors are devices that acquire observations from the surrounding environment. Those could include basic proximity sensors that are ultrasonic, laser or infrared based. Vision system, i.e. camera, also is a common sensing system. Sensors consume energy in proportion with the rate of observations. So, generally when considering sensors energy, the decision variable would be the rate of sensing (e.g. frame rate in a camera). A linear relation can sufficiently model energy versus rate.

- Microcontrollers & Computer: these components are responsible of controlling all processes of the robot. Energy consumption of computers depends on the execution of the program. However, the specific relation to consumption is complex. Complexity is due to the inherent complexity of how microprocessors handle algorithms and programs. So, when considered, it is either dropped or included as constant value.

IV. RELATED WORK

Here in this section, a quick overview of different examples of work that are done in literature. I do not claim that works reviewed here are the only ones. However, most works here have much of attention for the problem of Energy Efficiency in Mobile Robots.

In [3], energy consumption enters the problem as limit constraint. Actually, the problem is analyzed for both time and energy as constraints. As said earlier, time and energy are conflicting objectives. Here the problem tries to maximize the distance covered with time and energy constraints. An energy constraint resembles the energy stored in the battery. The problem is formulated as in (4) to find the optimal velocity profile for the robot.

$$max \int_0^{\tau} v(t)dt$$

$$s.t.$$

$$0 \leq t \leq \tau$$

$$\int_0^{\tau} p\big(v(t)\big)dt \leq \mathcal{E}$$

$$0 \le v(t) \le v_m \quad (4)$$

With $v(t)$ as the velocity of the robot. $p(v(t))$ is the power model of the robot with respect to the velocity. And τ, $\mathcal{E}$ and v_m are the time limit, energy (battery) limit and maximum velocity of the robot respectively. With complete derivation of solution in [3], an optimal velocity is to have the velocity as constant. However the value of this constant value depend on the values of τ, $\mathcal{E}$ and other factors. The constant velocity would be chosen according to rules of:

if $\mathcal{E} \le \tau p(v_o)$, the speed is v_o;
if $\mathcal{E} \ge \tau p(v_m)$, the speed is v_m;
if $\tau p(v_o) \le \mathcal{E} \le \tau p(v_m)$, the speed is v_1 $(v_o \le v_1 \le v_m)$,
at which the power $p(v_1)$ is equal to $\mathcal{E}/\tau$. (5)

With

$$v_0 = \arg\min_{v} \{\frac{p(v)}{v}\} \quad (6)$$ ■

In [4], case study of how mobile robots consume energy. Here we mention only two issues. In order to improve energy efficiency of motion of mobile robots, we can develop the strategies of:

- Dynamic Power Management: this concept is to program the robot computer to organize power management across all components of the robot. An example is to have the idle components shut off. But to have this done properly, a timeout rule should be maintained to avoid rapid switching on and off which consume a great deal of energy.
- Real-time Scheduling: here, the computer would be responsible of arranging different task according to some rule. In relation to energy, the scheduling is done according to duration of the task. So, higher priorities will be given to shorter and faster tasks. ■

Another set of work done in investigating energy-minimization are work of Kim and Kim [5, 6]. Both papers talk about the same problem. They consider a differential-driven wheeled mobile robot. The circuit driving the wheels motors is shown in figure 3.

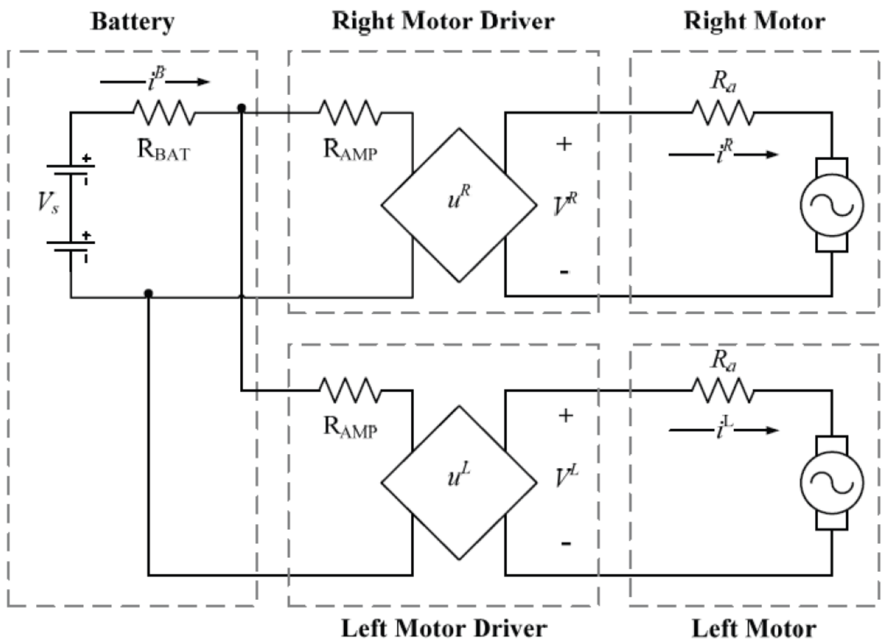


Figure 3: Motor Driver Circuit

A complete derivation of the problem is done in the papers. The optimization cost function considered includes also wheel friction and inertia along with the loss-minimization objective. So the total energy consumption considered can be set as

$$E_W = \underbrace{R_a \int_{t_0}^{t_f} \mathbf{i}^T \mathbf{i} \mathrm{d}t}_{\text{Energy dissipated in motor's armature resistance}} + \underbrace{F_v \frac{K_b}{K_t} \int_{t_0}^{t_f} \mathbf{z}^T \mathbf{T}_q^{-T} \mathbf{T}_q^{-1} \mathbf{z} \mathrm{d}t}_{\text{Energy dissipated due to wheels friction}} + \underbrace{\frac{K_b}{K_t} \int_{t_0}^{t_f} \dot{\mathbf{z}}^T \mathbf{T}_q^{-T} \mathbf{J}^T \mathbf{T}_q^{-1} \mathbf{z} \mathrm{d}t}_{\text{Kinetic Energy, wheels Inertias}}$$

With $\mathbf{z} = \begin{bmatrix} v \\ \omega \end{bmatrix}$ as v & ω are the translational and rotational velocities of the robot respectively. $\mathbf{T}_q$ is the transformation between wheels motion and robot motion. $\mathbf{i}$ is the current that drive the right and left motor. $\mathbf{J}$ is the inertia matrix of the motors. Other parameters related to values of armature resistance and friction coefficient and torque constant. As you can see, the relation between the wheels motion and robot motion is only the kinematic relation. So the problem becomes an optimization problem of finite horizon with constraints on control input limits. According to [5], the solution is proven analytically and experimentally to better than when only considering the loss-minimization due to dissipation of energy in armature resistance inside the motor. ■

Another interesting work of Kim & Kim [7] is in designing velocity profile in 3-step fashion. Considering a straight line motion, a velocity profile of typical 3-step is shown in figure 4.

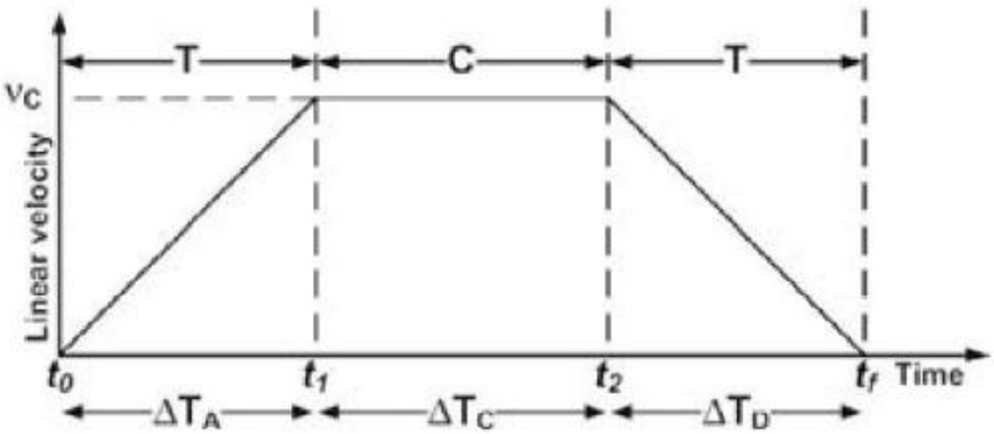


Figure 4: typical 3-step velocity profile

In above figure, subscripts A, C and D correspond to Acceleration, Cruise and Deceleration respectively. So it is obvious that for any stage of motion there will be an acceleration phase followed by constant cruise speed and ending by deceleration to zero. In [7], the optimization problem is formulated as in (7).

$$
\begin{aligned}
&\min E = E_A(\Delta T_A, u_A) + E_C(\Delta T_C, u_C) + E_D(\Delta T_D, u_D) \\
&\text{subject to } (1)\ \Delta T_A + \Delta T_C + \Delta T_D = t_f \\
&\qquad\qquad\quad (2)\ \Delta x_A + \Delta x_C + \Delta x_D = x_f \\
&\qquad\qquad\quad (3)\ v(t_0) = v(t_f) = 0 \\
&\qquad\qquad\quad (4)\ -1 \leq u_A, u_C, u_D \leq 1
\end{aligned}
\tag{7}
$$

With subscripted 'E' of energy consumed for each phase of the profile. Subscripted 'u' are control inputs to be decided for each phase. x_f is the final desired distance. The rest of variables are obvious. In [7], different kinds of accelerrations and decelerations are investigated (in above figure are all linear kind). The optimization problem is solved numerically with a search algorithm explained in paper. The paper claim to have a most efficient solution with efficiency of 30%. ■

Sergaki *et al.* [8] go deep in analyzing DC motor electromechanical losses. The energy consumed is fomulated as

$$
P_{\text{loss}} = \underbrace{i_a^2 R_a}_{\substack{\text{Armature}\\ \text{copper}}} + \underbrace{i_e^2 R_e}_{\substack{\text{Field}\\ \text{copper}}} + \underbrace{(C_1\omega^2 + C_2\omega)\varphi_e^2}_{\substack{\text{Armature}\\ \text{iron}}} + \underbrace{C_3\omega^2 i_a^2}_{\text{Stray load}} + \underbrace{2v_b i_a}_{\substack{\text{Brushes}\\ \text{contact}}} + \underbrace{f_\ell(\omega)}_{\text{friction}}
$$

You can see that from above complex equation that many components are considered. Detailed analysis of parameters is left for reader to see in the original paper. Along with the above energy evaluation the optimization problem will include the dynamic equations of the DC motor. Here the energy consumption is only studied for the DC motor. ■

Another interesting work was done by Sun & Reif [9]. An energy-minimization is considered for motion of mobile robots on terrains. Map of the terrains are captured and analyzed via satellite images of the area.

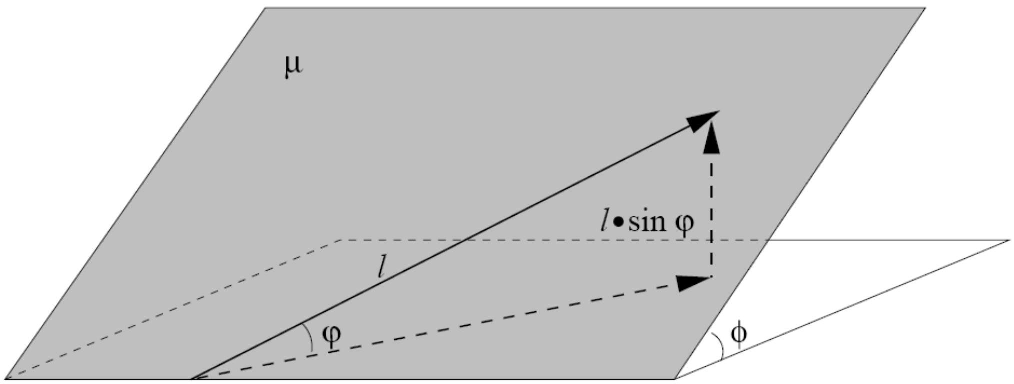


Figure 5: one face of a terrain

The terrain consists of multiple 'faces'. An example of a face is shown in figure 5. Imagine that the head of the vector in the figure is the location of the robot. With μ being the friction coefficient of this face, an energy loss (consumption) can be evaluated as

$$E = mg(w + \sin(\varphi))l \tag{8}$$

With the 'weight' $w = \mu \,.\cos(\phi)$. m is the mass of the robot. Other parameters are obvious from above figure. So, the whole terrain is discretized by several points on faces boundaries. So, these points if considered as 'vertices', links between them will form 'edges' that will construct a 'graph' with 'weighted' edge. As can be seen, the weight of each edge of the graph will depend on above (8). The weight consists of the effect of both face friction and gravity. Different values of φ, ϕ produce different types of faces with special considerations. Some values make a face inadmissible, for example. Also, by the same notion, other values of that make $(w + \sin(\varphi)) < 0$ means that the robot will roll down and accelerate and to gain energy. This situation will produce of equivalent braking energy. Above special situation and other normal ones will affect the values of weights of terrain graph's edges. The optimization problem is formulated to find an optimal path with considerations of path energy through the graph weights. An upper bound on number of edges is employed also. Other details of the optimal search are found in the paper [9]. ■

In Khoukhi's [10], a large paper is discussing the problem of optimization in time and energy as cybernetic problem. An offline programming is done. The paper discusses a step-by-step methodology of the problem. However, the part of designing motion and optimizing the energy is formulating the problem as Nonlinear Programming problem. A typical cost function is used (9).

$$\int_{t_0}^{t_f} \{C(t)RC^T(t) + \mu\} \tag{9}$$

With $C(t)$ is the torques applied; and μ is a weight on time optimization. The cost function is similar to (3). However, [10] go further to include multiple constraints:

$$\left\{\begin{aligned} x_{i+1} &= x_i + h_i v_i \cos(\theta_i) \\ y_{i+1} &= y_i + h_i v_i \sin(\theta_i) \\ \theta_{i+1} &= \theta_i + h_i \theta_i \\ v_{i+1} &= v_i + \frac{h_i}{rM}(C_1^i + C_2^i) \\ \omega_{i+1} &= \omega_i + \frac{h_i a}{2rJ}(C_1^i - C_2^i) \end{aligned}\right\} \text{Discrete dynamics}$$

$$\left.\begin{aligned} x_{\min} < x_i < x_{\max} \\ y_{\min} < y_i < y_{\max} \end{aligned}\right\} \text{Space limits}$$

$$\left.\begin{aligned} v_{\min} < v_i < v_{\max} \\ \omega_{\min} < \omega_i < \omega_{\max} \end{aligned}\right\} \text{To prevent skidding}$$

$$\left.\begin{aligned} C_{1\min} < C_{1i} < C_{1\max} \\ C_{2\min} < C_{2i} < C_{2\max} \end{aligned}\right\} \text{Torques limits}$$

Further details of parameters are explained thoroughly in original paper. The offline planning is determined by solving the Nonlinear Programming problem. In [10], an algorithm called "Discrete Augmented Lagrangian (DAL)" is used to solve the optimization problem. ■

Qin *et al.* [11] consider a Car-Like Robot. A model of the motion is set as

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = v\tan\phi, -\phi_{\max} \le \phi \le \phi_{\max}$$
$$\dot{\phi} = \alpha, \alpha_{\max} \le \alpha \le \alpha_{\max} \tag{10}$$

With *x, y* as robot position coordinates; θ is orientation of the robot; *v* is a *fixed* velocity of the robot. With ϕ is the steering angle. α will perform as the control input. The cost function is set as

$$J = \int_{t_0}^{t_f} |\alpha|\, dt \tag{11}$$

According to [11], authors solved the problem with a Bang-Bang control. The solution is discussed in detail in the original paper. ■

Ancenay and Maire in [12] chose motion planning as their example to test the quadratic programming algorithm. The motion model of the robot is put as simple as possible making position as summation increments of velocity; and velocities as summation of increments in acceleration. The cost function is set as

$$\min \sum_{k=1}^{n} \left\| A^k \right\|^2 \tag{12}$$

In (12), A^k is the acceleration at increment *k.* The paper [12] can be just considered as a quadratic programming optimization with mobile robot as an example. Solution is further explained in the paper. ■

An early paper in energy efficiency in mobile robots is [13] by Duleba *et al.* In this high mathematical paper, a Newton method is employed to nonholonomic robots in energy-efficient way. ■

V. FOCUS WORK

In this section, we begin a focused study of a specific work. One of the well-done works in Energy Efficiency in Mobile Robots is that of Mei *et al.* [1-4]. In his thesis [1], Mei studies the problem of energy efficiency in mobile robots. The study includes motion planning along with deployment strategies of multi-robot systems. Throughout his work, the power models used are identified experimentally for two robots.

In this report, the work done in [2] will be used as the main source of analysis and results. This paper is selected because of its relevance to the field along with its practicality. Also, [2] analyzes energy efficiency as integrated study of planning and direct actuation of the motors. These is of great importance as other studies, as mentioned earlier, are either studying only optimal planning with no regard of motor consumption; or only analyzes 'optimal' control of motor motion The paper [2] studies energy-efficient motion planning for area coverage. So efficiency is considered to be as shown in (13).

$$Efficiency = \frac{Covered\ Area}{Energy\ Consumed} \tag{13}$$

The power model of the robot is identified experimentally as $P(\omega, \alpha)$ with ω & α are motor angular velocity and angular acceleration, respectively. So, the complete power consumption of a multi-motor robot can be formed in (14).

$$\sum_{i=1}^{K} P\left(\frac{v_i(t)}{r}, \frac{1}{r}\frac{dv_i(t)}{dt}\right) \tag{14}$$

With K the number of motors in the robot. And $v_i(t)$ is velocity of motor i with radius r. According to the paper, generally, motor energy consumption is related significantly to the angular velocity rather than the angular acceleration of the motor. So the angular acceleration can be dropped throughout the analysis. In the study, a Palm Pilot Robot Kit (PPRK) is used to conduct the experiments. The PPRK is developed by Manipulation Lab at the Robot Institute, Carnegie Mellon University. You can see a photo of the robot in figure 6. The PPRK consists of 3 servomotors. All the motors are the same. Experiments are performed to estimate the power model of the motor. A 6^{th}-order polynomial is fitted for the data. Equation (15) shows the polynomial of the power as a function of the angular velocity.

$$\begin{aligned} P_m(\omega) = \ & 4.216 \times 10^{-5}\omega^6 + 1.387 \times 10^{-4}\omega^5 - 6.777 \times 10^{-3}\omega^4 + 4.238 \times 10^{-2}\omega^3 \\ & -1.016 \times 10^{-1}\omega^2 + 1.178 \times 10^{-1}\omega + 1.695 \times 10^{-1}. \end{aligned} \tag{15}$$

The model above only takes into mind the effect of velocity to the energy consumption of the motor. You can see in figure 7 the plot of the power versus the angular velocity. As the robot consists of 3 motors, then the total power of the robot is computed the same as eq. (14) with $K=3$ and negligible acceleration. For a period of time T in operation, the energy consumed is calculated as

$$Energy = \int_0^T Power\, dt \tag{16}$$

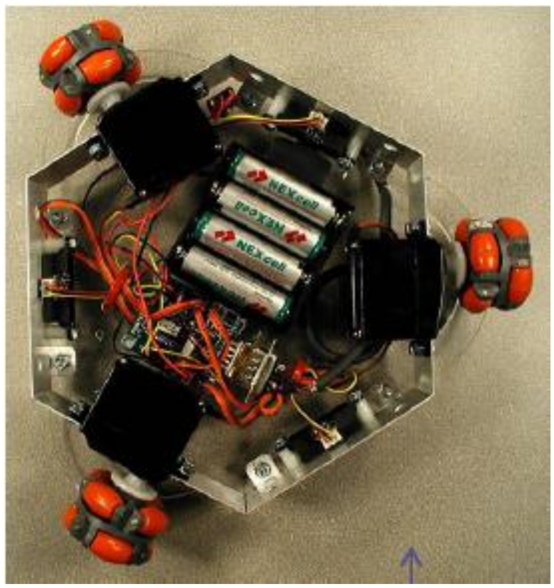


Figure 6: PPRK Robot

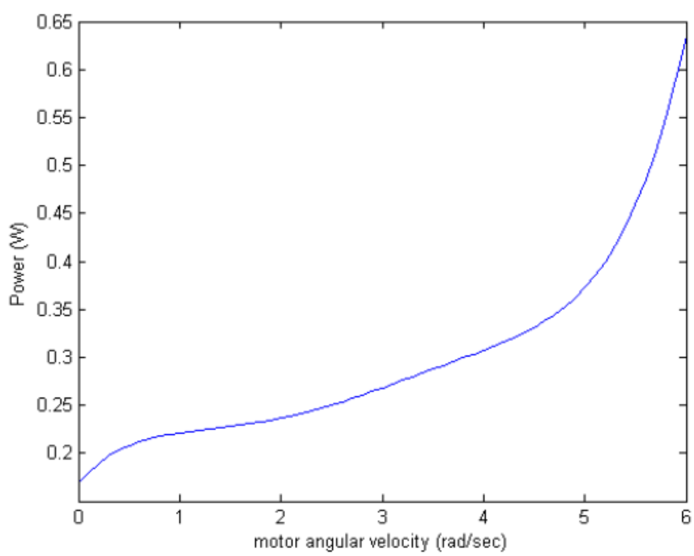


Figure 7: Power vs. Angular Velocity

A kinematic relation is employed to relate wheel rotation to robot motion. You can see a schematic of the robot in figure 8. As seen in figure, $V_x, V_y,$ corresponds to the velocity on x-direction, velocity on y-direction, and angular velocity of the robot body, respectively. Also, V_1, V_2, V_3 corresponds to the linear velocities of the motors. The robot structure is encircled in shape. With radius of b, the Jacobian transforming the robot velocity into motor velocities is shown in (17).

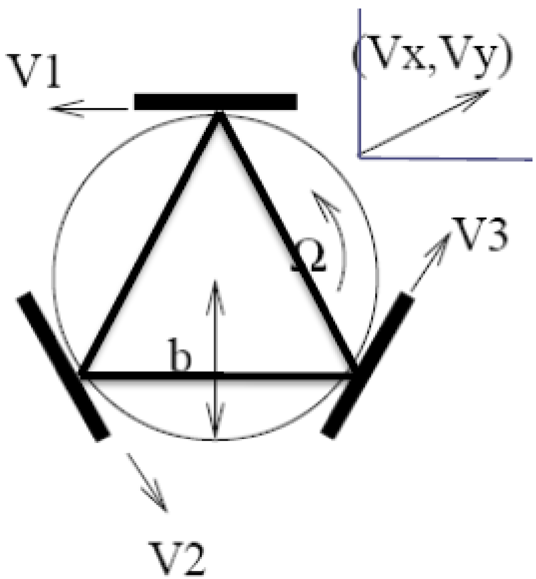


Figure 8: Robot Schematic

The relation in (17) gives a direction transformation (static) ignoring the dynamics of the robot body. From the structure of the robot and also from the mathematical relation in (17), you can see that the robot is Omnidirectional, i.e. it can change its direction independent of the position (x, y).

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & b \\ 1/2 & -\sqrt{3}/2 & b \\ 1/2 & \sqrt{3}/2 & b \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ \Omega \end{bmatrix} \tag{17}$$

The problem statement here is to investigate Energy Efficiency of a mobile robot for Open Area Covering using different methods (namely, Scan-lines, square spirals, and spirals).

Pictorial view of these methods is shown in figure 9. The following section will discuss the motion and energy of the robot for the three methods:

- Scanlines: V-A
- Square Spirals: V-B
- Spirals : V-C

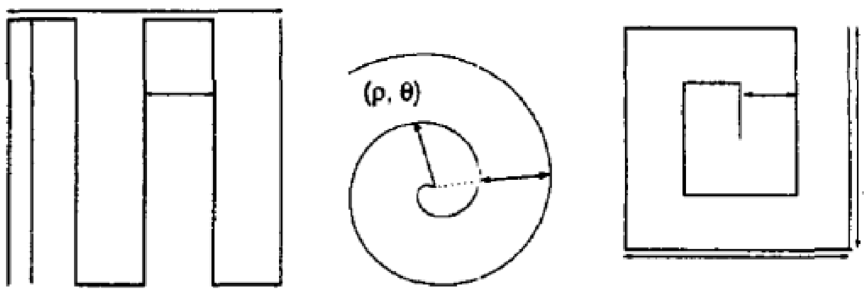


Figure 9: Area Coverage Strategies

A) Scanlines Area Covering

Let us assume that at straight lines the robot moves on a constant speed of S. to reach this velocity at each segment, an acceleration and deceleration phases occurs at a magnitude of A. figure 10 shows clearer view of the method. The area is divided into several segments with length of h and width $2l$ for each. At each corner, the robot stops (after deceleration) and then turns 90° with motion of constant rotational speed of Ω and rotational acceleration and deceleration of Λ to reach the desired. So, for some n, we will have $n+1$ segments of length h, n segments of width $2l;$ and $2n$ rotations. So, the total area becomes $(2l+h)(2l+2nl)$.

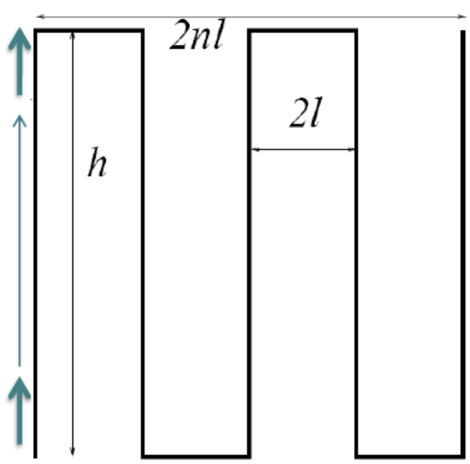


Figure 10: Scanlines Area Covering

So to cover the area in scanlines, the motion will consist of these stages: 1) acceleration on beginning of each segment, 2) deceleration on end of each segment, 3) constant speed motion in each segment, 4) acceleration and deceleration on rotational motion, 5) constant rotation speed on rotations. For each of the stages, the energy consumed will be computed to get the total energy of the motion.

- ***For constant speed motion*** in segments, the robot body speed can be evaluated as $[0 \quad S \quad 0]^T$. So with eq. (17), motor speeds are computed. Power is then computed via eq. (14). As in this motion the power is constant, energy computed by (16) becomes

$$E_1 = P_S \cdot (\frac{[(n+1)\left(h - {S^2}/{A}\right) + n\left(2l - {S^2}/{A}\right)]}{S}$$

  With P_S is the power, and the rest of the term is the **time** taken to accomplish the motion for all segments.

- ***For acceleration and deceleration*** in segments, robot speed is not constant. So, the speed is evaluated as $[0 \quad At \quad 0]^T, [0 \quad S - At \quad 0]^T$ for acceleration and deceleration respectively. To compute the energy,

$$E_2 = (2n+1) \int_0^{S/A} P_A + P_D \, dt$$

  With P_A, P_D are power at acceleration and deceleration stages that are repeated $(2n+1)$ times.

- By the same notion, energy can be computed ***for rotation*** stages as

$$E_3 = P \cdot \frac{\left[(2n)(90 \; - \; {}^{2}/{\Lambda})\right]}{} + 2n \int_0^{/\Lambda} P_\Lambda \, dt$$

  With P is the power for the constant speed rotation, and P_Λ as the power for angular accelerations and decelerations; all with the corresponding speed and acceleration values.

So the **total energy** is calculated as $E_1 + E_2 + E_3$. So, the energy efficiency for area covering using scanlines can be computed by eq. (13).

B) Square Spirals

Here, we analyze the problem when motion would be done using square spirals methods. In this motion, the area is almost the same as motion in scanlines with the only difference of the changing segment length. Figure 11 shows the method. The total area here is only restricted to square area of $(2nl+2l)^2$.

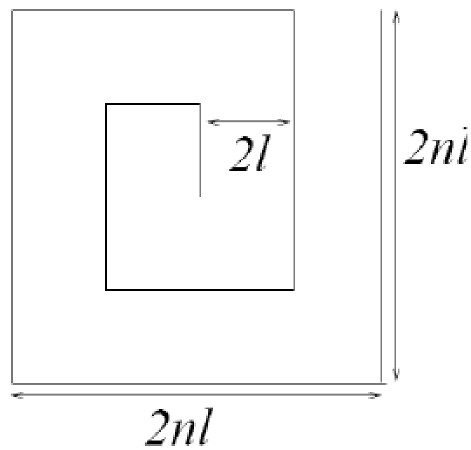


Figure 11: Area Covering in Square Spirals

Actually, to calculate energy consumed, the problem only changed slightly as of the problem in scanlines. Namely, energy consumed for accelerations and decelerations and the whole rotation stages are the same. If you noticed, motion is not different in those stages. So, the only difference is the energy consumed when moving in constant speed along the straight segments. The energy consumed is

$$E_1 = P_S \cdot \frac{1}{S}\left[\left(2nl - {S^2}/{A}\right) + 2\sum_{i=0}^{n}\left(2il - {S^2}/{A}\right)\right]$$

The only difference is in the change of length of segments that deduces into change in time taken for each segment. By the way, the energy efficiency can be computed.

C) Spirals

In this section, energy efficiency is analyzed for motion in spiral motion. In [2], the authors did not give much detail about their work. So, the analysis below is done independently. Figure 12 shows the motion of the robot along the spiral path.

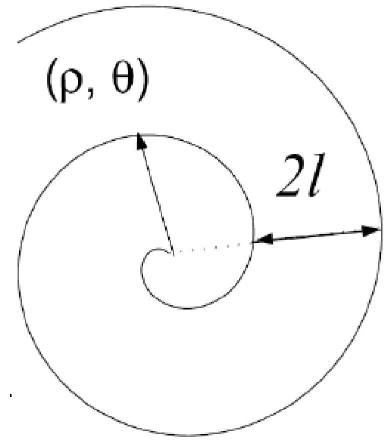


Figure 12: Spiral Area Covering

In a spiral motion the 'radius' of the curve is continually changing. To have an increase of 2l for each revolution, then $\rho = \frac{l}{\pi}\theta$. So, the path curve is now depends only on motion revolution θ. To compute the total area covered for some motion, a final revolution θ_F will result in an area of

$$Area = 2l^2 + \frac{\pi l^2}{3} + \frac{l^2\theta_F^2}{\pi}$$

To calculate the power consumed when the robot moves in a spiral path, then speeds of the robot is changing with time. Moreover, the angular velocity of the robot body treats the problem as rigid body. So let us assume that the robot body at some time is at position of

$$p(\rho, \theta) = \rho\theta$$

Then we can have the location of the robot as

$$x = \rho \cdot \cos(\theta)$$

$$y = \rho \cdot \sin(\theta)$$

So, with $\rho = \frac{l}{\pi}\theta$, we can have robot's linear speeds of

$$V_x = \dot{x} = \frac{l}{\pi}[\cos\theta - \theta\sin\theta]\dot{\theta}$$

$$V_y = \dot{y} = \frac{l}{\pi}[\sin\theta + \theta\cos\theta]\dot{\theta}$$

For convenience, let us assume the total linear speed of the robot is $S = \sqrt{V_x^2 + V_y^2}$, then, rate of change in curve revolution is

$$\dot{\theta} = \frac{\pi S}{l \cdot \sqrt{1 + \theta^2}}$$

So, from above relation we can have V_x, V_y for any θ.

For the angular speed of the robot body, we need to utilize the concept of curvature. Here we have a spiral path, so the 'curvature' of the path changes with position. It is consistent to say that angular speed of the robot body reduces as the robot gets far from origin, i.e. as θ increases. To measure the angular speed for any curved path, it is calculated as

$$= \frac{S}{R} = KS$$

With

$$K = \frac{\pi(\pi^2 \rho^2 + 2l^2)}{(\pi^2 \rho^2 + l^2)^{3/2}}$$

With R is the radius of curvature and K is called curvature. So, to compute power consumed for a spiral motion for this robot, the speed vector is set as $[V_x \quad V_y \quad]^T$. So at the end, the total energy is calculated as

$$E_{sp} = \int_0^T P_{sp} \, dt$$

With P_{sp} is power consumed for the provided speeds. To be more unproblematic, the energy can be evaluated as θ changes, but a change of variables is needed as $\dot{\theta} = \frac{d\theta}{dt}$. So, the integration becomes

$$E_{sp} = \int_0^{\theta_F} \frac{P_{sp}}{\dot{\theta}} \, d\theta$$

VI. SIMULATION AND RESULTS

Here in this brief section, simulations are done to reproduce the results attained by [2]. Also, further modifications of the problem is tested, namely robot parameters and robot kinematic model.

First, with explained procedure in section V, all three methods are tested for their efficiency, with n is a simulation control parameter of area. Also, values of *S=0.08, A=0.2,* *Ω=2/3*, *Λ=5/3* are set as default. For robot specification, PPRK have radius b=0.12m with wheel radius $r=0.02m$. And as needed in simulations, default values of $l=0.3m$ and $h=0.8m$ are used. So, for different values of areas, figure 13 have a plot of the efficiency of the three methods **(red: square spirals, blue: spirals, black: scanlines).**

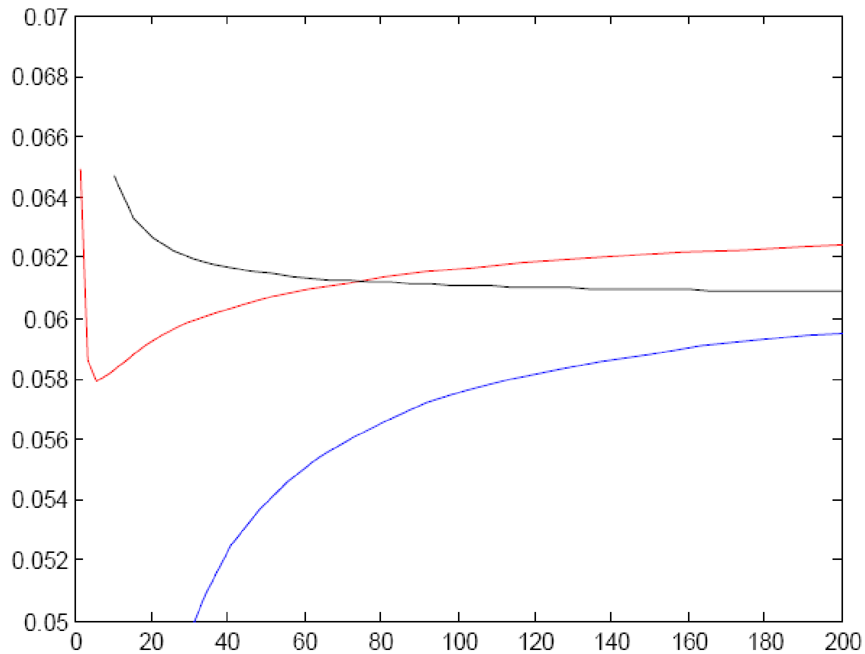


Figure 13: Efficiency vs. Area

For scanlines, different values of segment length h are tested for a constant area of 100. The efficiency plot is shown in figure 14.

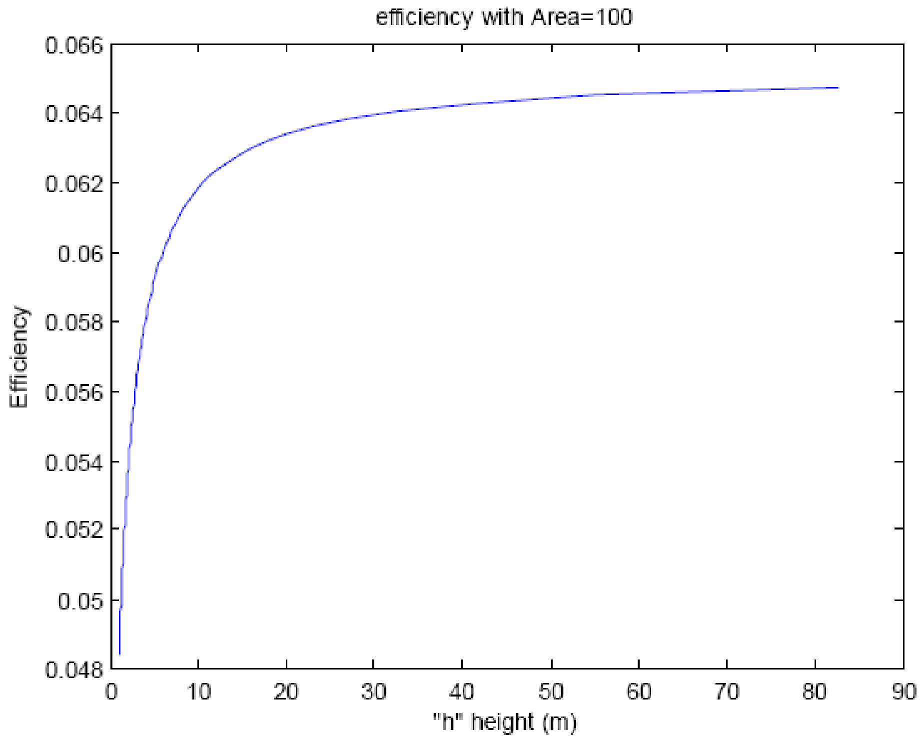


Figure 14: Efficiency vs. segment length

Another analysis is done on the effect of different speeds on the efficiency. So, for a constant area of 100, efficiency is computed for different values of *S*. figure 15 shows that the optimal speed of this robot is about 0.08.

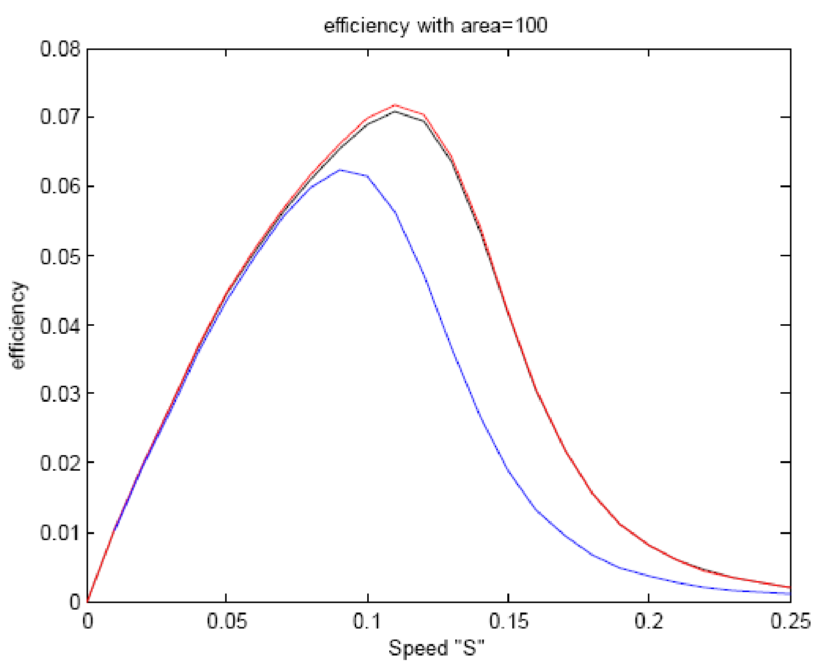


Figure 15: Efficiency vs. Speed

However, for different speeds, time taken to cover the area changes. So, in figure 16, time (sec) is plotted versus the speed.

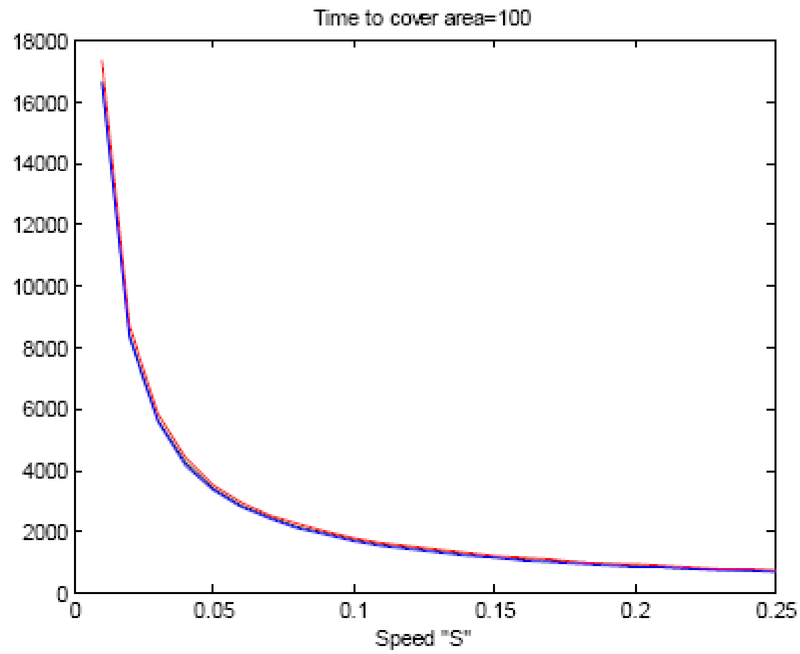


Figure 16: Time vs. Speed

To have another analysis of the robot at hand, change in some parameter of the robot may change the efficiency. So, a wheel radius of 0.03m is tested. So for different speeds, efficiency is shown in figure 17. As expected, higher efficiency is met with higher speeds.

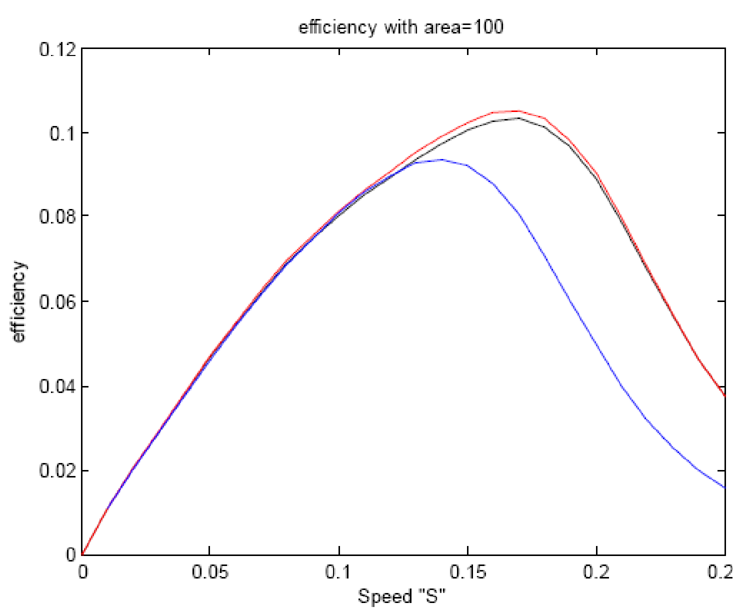
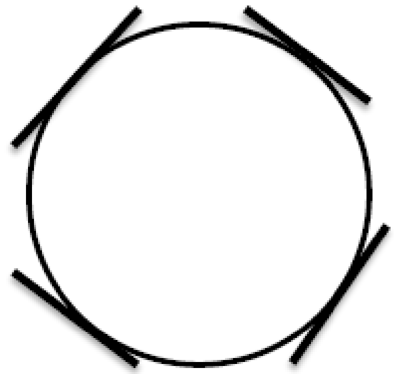


Figure 17: Efficiency with different wheel radius

Another test is done for different robot structure. Let us assume a robot as shown in figure 18. The Jacobian is shown also.



$$J = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} & b \\ 1/\sqrt{2} & 1/\sqrt{2} & b \\ -1/\sqrt{2} & -1/\sqrt{2} & b \\ 1/\sqrt{2} & 1/\sqrt{2} & b \end{bmatrix}$$

Figure 18: 4-wheel robot with its Jacobian

The increased number of motors will definitely affect the efficiency. In figure 19, the plot is shown.

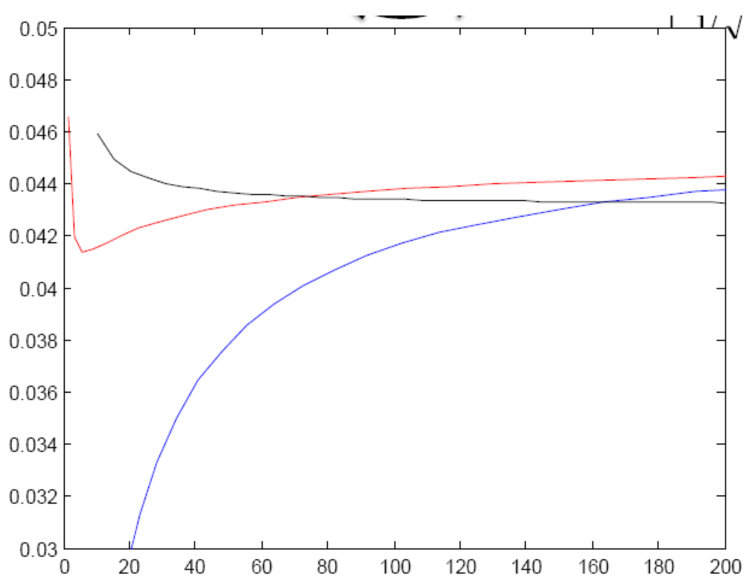


Figure 19: Efficiency for 4-wheel robot

VII. Future Directions & Conclusion

In this brief section, general discussion about the topic and focused work is shown. Throughout the mentioned problem of Energy Efficiency in Motion Planning for Mobile Robots, wide range of approaches is tested. Extension to the work is to:

- Test other covering methods
- However, a better work would be good if the problem is formulated as an Optimization Problem for any given area (with/without obstacles, terrain, etc).
- Maybe also to analyze the problem with dynamic behavior of the motor or the robot.
- Online energy optimization should be also investigated.

Other general comments:

- Motor energy losses is studied well but still only investigated for basic motions (i.e. straight lines)
- In the same time, no analytic solution is studied to relate energy consumption to path planning. Most studies are just special cases.
- Energy (power) models are mostly identified experimentally. So, for each problem and for each robot system, power models would be investigated independently.
- The field of energy efficiency of mobile robots is still relatively new. Up till now, to the knowledge of the author, ***no solid theoretic foundation*** of the field is built yet. Specifically, there is no way found to explicitly figure a relation between energy consumption and paths. So, a theoretic foundation is needed.

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Appendix: MATLAB Code

1) Subroutine: **rob_power.m** to calculate power for given Speeds, Jacobian and robot parameters

```
function pow1=rob_power(V,J,r)

% V=[Vx;Vy;W];
v=abs((J*V)./r);

power=4.216e-5*v.^6+1.387e-4*v.^5-6.777e-3*v.^4+4.238e-2*v.^3-1.016e-
1*v.^2+1.178e-1*v+1.695e-1;
pow1=sum(power);
```

2) Main program

```
clear all
clc

b=0.12;
r=.02;

Jcb=[-1 0 b;.5 -sqrt(3)/2 b;.5 sqrt(3)/2 b];

%Jcb=[1/sqrt(2)  1/sqrt(2)  b;-1/sqrt(2)  1/sqrt(2)  b;-1/sqrt(2)  -1/sqrt(2)
b;1/sqrt(2) 1/sqrt(2) b];

%% scan lines

h=8;
n=1:100;
% n=19;
l=.3;
% S1=0:.01:.25;
% for j1=1:length(S1)
for j=1:length(n)

% j=j1;

% hh(j)=(100-4*n(j)*l^2-4*l^2)/(2*n(j)*l+2*l);
% h=hh(j);

S=.08;
% S=S1(j1);
A=.2;
W=2/3;
Ap=5/3;


V=[0;S;0];
% v=(Jcb*V)/r;
Ps=rob_power(V,Jcb,r);

Ts=((n(j)+1)*(h-S^2/A)+n(j)*(2*l-S^2/A))/S;

Es=Ps*Ts;

A1(j)=(2*n(j)*l+2*l)*(h+2*l);
```

```
% eff1=As/Es;

% t=0;
dh=0.01;
t=0:dh:S/A;
Pa=0;
Pd=0;

for i=1:length(t);
Pa=Pa+rob_power([0;A*t(i);0],Jcb,r)*dh;
Pd=Pd+rob_power([0;S-A*t(i);0],Jcb,r)*dh;
end

Ea=(2*n(j)+1)*Pa(end);
Ed=(2*n(j)+1)*Pd(end);

Pw=rob_power([0;0;W],Jcb,r);

dh=0.01;
t=0:dh:W/Ap;
PAp=0;
PD=0;
for i=1:length(t);
PAp=PAp+rob_power([0;0;Ap*t(i)],Jcb,r)*dh;
PD=PD+rob_power([0;0;W-Ap*t(i)],Jcb,r)*dh;
end

Tw=((2*n(j))*(pi/2-W^2/Ap))/W;

Ew=Pw*Tw+2*n(j)*(PAp+PD);

eff1(j)=A1(j)/(Es+Ea+Ed+Ew);

% T1(j1)=A1(j)/S;
% T1(j1)=Ts+Tw+(2*S/A)+(2*W/Ap);

end
% end
%% square spirals

n=1:100;
% n=16;
l=.3;
% S1=0:.01:.25;
% for j1=1:length(S1)
for j=1:length(n)

S=.08;

% S=S1(j1);
A=.2;
W=2/3;
Ap=5/3;

V=[0;S;0];

A2(j)=(2*n(j)*l+2*l)^2;

Ps=rob_power(V,Jcb,r);

Ts=((2*n(j)*l-S^2/A)+2*sum((2*(1:n(j)).*l-S^2/A)))/S;
```

```
Es=Ps*Ts;

dh=0.01;
t=0:dh:S/A;
Pa=0;
Pd=0;

for i=1:length(t);
Pa=Pa+rob_power([0;A*t(i);0],Jcb,r)*dh;
Pd=Pd+rob_power([0;S-A*t(i);0],Jcb,r)*dh;
end

Ea=(2*n(j)+1)*Pa(end);
Ed=(2*n(j)+1)*Pd(end);

Pw=rob_power([0;0;W],Jcb,r);

dh=0.01;
t=0:dh:W/Ap;
PAp=0;
PD=0;
for i=1:length(t);
PAp=PAp+rob_power([0;0;Ap*t(i)],Jcb,r)*dh;
PD=PD+rob_power([0;0;W-Ap*t(i)],Jcb,r)*dh;
end

Tw=((2*n(j))*(pi/2-W^2/Ap))/W;

Ew=Pw*Tw+2*n(j)*(PAp+PD);

eff2(j)=A2(j)/(Es+Ea+Ed+Ew);

% T2(j1)=A2(j)/S;
% T2(j1)=Ts+Tw+(2*S/A)+(2*W/Ap);

end
% end
%% spirals

% n=1:100;
% S1=0:.01:.25;
% for j1=1:length(S1)
TH=(0:30).*pi;
% TH=59;

l=.3;

PP=[];

for k=1:length(TH);

A3(k)=2*(l^2)+(pi*(l^2))/3+((l^2)*(TH(k)^2))/pi;


S=.08;
% S=S1(j1);
A=.2;
Wp=2/3;
% Wp=1/3;
Ap=5/3;

% V=[0;S;0];
dt=.05;
```

```
t=0:dt:S/A;

Pasp=0;
Pdsp=0;
% tho=zeros(1,length(t));
% for i=1:length(t);
%
% thd(i)=(pi*(A*t(i))/l)/(1+tho(i)^2)^(1/2);
%
% % tho(i)=tho
%
% Vx(i)=(l/pi)*thd(i)*(cos(tho(i))-tho(i)*sin(tho(i)));
%
% Vy(i)=(l/pi)*thd(i)*(sin(tho(i))+tho(i)*cos(tho(i)));
%
% tho(i+1)=tho(i)+thd(i);
%
% ro(i)=(l/pi)*tho(i);
%
% K=pi*((pi^2)*(ro(i)^2)+2*(l^2))/(((pi^2)*(ro(i)^2)+(l^2))^(3/2));
%
% W(i)=K*sqrt(Vx(i)^2+Vy(i)^2);
% % W2(i)=K*sqrt((S-Vx(i))^2+(S-Vy(i))^2);
%
%
%
% Pasp=Pasp+rob_power([Vx(i);Vy(i);W(i)],Jcb,r)*dt;
% % Pdsp=Pdsp+rob_power([(S-Vx(i));(S-Vy(i));W2(i)],Jcb,r)*dt;
% end
%
% th0=tho(end);

tho=[];
Vx=[];
Vy=[];
W=[];
thd=[];
ro=[];

% tt=[];
dh=.05;
Tf=(l*(TH(k))^2)/(2*pi*S);
% Tf=TH(k)/Wp;
% tt=0:dh:Tf;
% th0=sqrt(2*pi*S*S/(l*A));
th0=0;
tho=(th0:dh:TH(k));
% ro=tho.*l./pi;
% tho=[];
% ro=[];
Psp=0;
W=[];

TT=0;
for kk=1:length(tho)

% tho(kk)=sqrt(2*pi*S*(tt(kk))/l);

% tho(kk)=2*pi*S/(l*Wp);

thd(kk)=(pi*S/l)/(1+tho(kk)^2)^(1/2);

Vx(kk)=(l/pi)*thd(kk)*(cos(tho(kk))-tho(kk)*sin(tho(kk)));
```

```
Vy(kk)=(l/pi)*thd(kk)*(sin(tho(kk))+tho(kk)*cos(tho(kk)));

Sv(kk)=sqrt(Vx(kk)^2+Vy(kk)^2);

ro(kk)=(l/pi)*tho(kk);

K=pi*((pi^2)*(ro(kk)^2)+2*(l^2))/(((pi^2)*(ro(kk)^2)+(l^2))^(3/2));

% K=(pi/l)*(tho(kk)^2+2)/(tho(kk)^2+1)^(3/2);

W(kk)=K*S;

% W=(2*l/pi)*tho(kk)*dh;

V=[Vx(kk);Vy(kk);W(kk)];

% dtt=(l*(dh^2)/(2*pi*S));

% dtt=(l*tho(kk)*dh)/(S*pi);

% dtt=(S*pi)/(l*tho(kk));

% dtt=dh;
%
% dtt=Tsp/length(ro);

% Tsp=(l*(TH(k))^2)/(2*pi*S);

dtt=dh/thd(kk);

TT=TT+dtt;

Psp=Psp+rob_power(V,Jcb,r)*dtt;
end

% tho=[];
% Vx=[];
% Vy=[];
% W=[];
% thd=[];
% ro=[];

dt=.05;
t=0:dt:S/A;

% tho=zeros(1,length(t));
% tho(1)=TH(k)-th0;
% for i=1:length(t);
%
% thd(i)=(pi*(S-A*t(i))/l)/(1+tho(i)^2)^(1/2);
%
% % tho(i)=tho
%
% Vx(i)=(l/pi)*thd(i)*(cos(tho(i))-tho(i)*sin(tho(i)));
%
% Vy(i)=(l/pi)*thd(i)*(sin(tho(i))+tho(i)*cos(tho(i)));
%
% tho(i+1)=tho(i)+thd(i);
%
% ro(i)=(l/pi)*tho(i);
%
% K=pi*((pi^2)*(ro(i)^2)+2*(l^2))/(((pi^2)*(ro(i)^2)+(l^2))^(3/2));
```

```
%
% W(i)=K*sqrt(Vx(i)^2+Vy(i)^2);
% % W2(i)=K*sqrt((S-Vx(i))^2+(S-Vy(i))^2);
%
%
%
% % Pasp=Pasp+rob_power([Vx(i);Vy(i);W(i)],Jcb,r)*dt;
% Pdsp=Pdsp+rob_power([(Vx(i));(Vy(i));W(i)],Jcb,r)*dt;
% end


Tsp=1;

% Tsp=dtt*kk;

PP(k)=(Psp+Pasp+Pdsp);
Esp(k)=PP(k)*Tsp;

eff3(k)=A3(k)/Esp(k);

% T3(j1)=A3(j)/S;
% T3(j1)=Tf;

end
% end
plot(A3,eff3)
axis([0 200 .03 .05])
hold
plot(A2,eff2,'r')
plot(A1,eff1,'k')

% figure
% plot(S1,eff1,'k')
% title('efficiency with area=100')
% ylabel('efficiency')
% xlabel('Speed "S"')
% hold
% plot(S1,eff2,'r')
% plot(S1,eff3)
%
% figure
% plot(S1,T1,'k')
% title('Time to cover area=100')
% ylabel('time (sec)')
% xlabel('Speed "S"')
% hold
% plot(S1,T2,'r')
% plot(S1,T3)
```