Term 081
KFUPM
EE 656: Robotics & Control
Research Project

Energy-Efficient Motion Control of Mobile Robots

Mohammad Shahab,
Systems Engineering dept.

Outline

- Energy Efficiency in Mobile Robots
- Energy Consumption in Mobile Robots
- Related Work
- Focus Paper
  - Problem Statement
  - System Models: Kinematic, Power
- Efficiency in Area Coverage:
  - Scanlines
  - Square spirals
  - Spirals
- Results (reproduced)
- Extra:
  - Different system parameters
  - Different Robot structure (other kinematics)
- Future Directions

Energy Efficiency in Mobile Robots

$$Efficiency = \frac{output}{input} = \frac{output \quad task}{energy \quad consumption}$$

- Consumption related to Power Source (Battery) life which is ***finite!***
- Goal:
  - To maximize ratio
  - To maximize battery life, Cost
  - To maximize task output: time, area or distance covered, etc AND
  - To minimize Energy Consumption

Energy Consumption in Mobile Robots

- To minimize the consumption in:
  1. Motors: *direct*
     - To minimize Energy Dissipation (loss).
     - Power model related to motor current driver circuit
  2. Motion: *indirect (ignoring motor losses)*
     - *To minimize Time, to minimize distance (discrete analysis)*
     - *Also, cost function: "input power"* $u(t)^2$
     - Produces 'optimal' trajectory
  3. Motors + Motion: *Integrated Analysis*
     - *Analyzing both trajectory & motor*
     - *Conflicts between time and energy*
  4. Auxiliary Sources:
     - sensors frequency
     - Microprocessor or Computer

Related Work (1)

- Focus paper ***later on*** in the presentation

- New paper 2009! From Purdue University:

Brief paper

Optimal solutions to a class of power management problems in mobile robots☆

Wei Zhang*, Yung-Hsing Lu, Jianghai Hu

Department of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47906, USA

A R T I C L E   I N F O

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Keywords:
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Scheduling of real-time systems
Low power design of embedded systems

A B S T R A C T

This paper studies an approach to minimize the power consumption of a mobile robot by controlling its traveling speed and the frequency of its on-board processor simultaneously. The problem is formulated as a discrete-time optimal control problem with a random terminal time and probabilistic state constraints. A general solution procedure suitable for arbitrary power functions of the motor and the processor is proposed. Furthermore, for a class of realistic power functions, the optimal solution is derived analytically. Interpretations of the optimal solution in the practical context are also discussed. Simulation results show that the proposed method can save a significant amount of energy compared with some heuristic schemes.

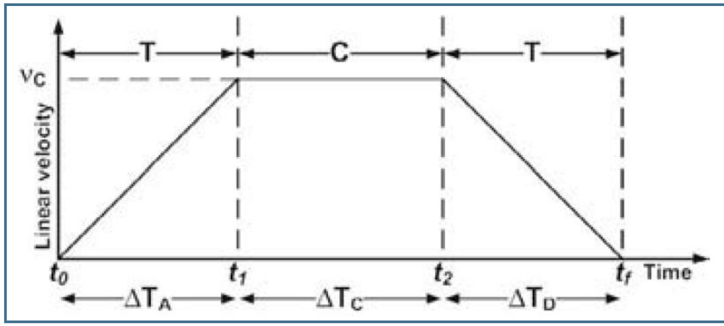
Related Work (2)

1. *Y.Mei et al "Deployment of Mobile Robots with Energy and Timing Constraints"*
   - Minimize time & maximize energy
   - Optimization problem with: distance cost & energy limit constraint
   - Solution: constant speed depending on energy and velocity limits
2. *Y.Mei et al "A Case Study of Mobile Robot's Energy Consumption and Conservation Techniques":* Power Management & Real-time Scheduling
3. *Kim & Kim, "Minimum-Energy Motion Planning for Differential-Driven Wheeled Mobile Robots" and "Minimum-Energy Translational Trajectory Generation for Differential-Driven Wheeled Mobile Robots":* Optimization on Motor

$$E_W = R_a \int_{t_0}^{t_f} \mathbf{i}^T \mathbf{i} \mathrm{d}t + F_v \frac{K_b}{K_t} \int_{t_0}^{t_f} \mathbf{z}^T \mathbf{T}_q^{-T} \mathbf{T}_q^{-1} \mathbf{z} \mathrm{d}t + \frac{K_b}{K_t} \int_{t_0}^{t_f} \dot{\mathbf{z}}^T \mathbf{T}_q^{-T} \mathbf{J}^T \mathbf{T}_q^{-1} \mathbf{z} \mathrm{d}t$$

4. *Kim & Kim, "Energy-Saving 3-Step Velocity Control Algorithm for Battery-Powered Wheeled Mobile Robot"*

$$\min E = E_A(\Delta T_A, u_A) + E_C(\Delta T_C, u_C) + E_D(\Delta T_D, u_D)$$

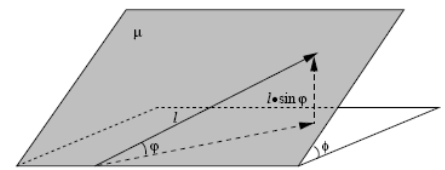


Related Work (2)

5. *Sergaki et al. "Optimal Robot Speed Trajectory by Minimization of the Actuator Motor Electromechanical Losses".* Analytic Motor power model

$$P_{\text{loss}} = i_a^2 R_a + i_e^2 R_e + (C_1\omega^2 + C_2\omega)\varphi_e^2 + C_3\omega^2 i_a^2 + 2v_b i_a + f_\ell(\omega)$$

6. *Z. Sun & J. Reif, "On Energy-minimizing Paths on Terrains for a Mobile Robot"*
   - *Terrain faces graph discretized into edges with weights*
   - *Search methods*



7. *Khoukhi, "Dynamic Modelling and optimal time-energy off-line programming of mobile robots: A cybernetic problem"***:** Nonlinear Programming with many constraints.

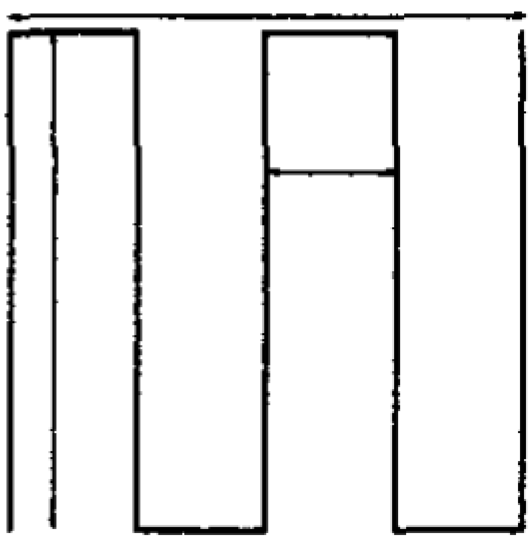
8. *Qin et al., "Optimal Trajectory generation for wheeled mobile robot":* Car-Like, limits on rotations,

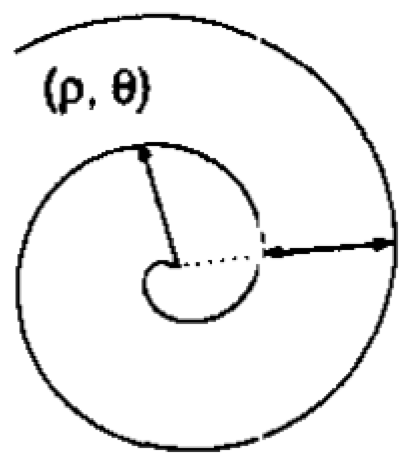
9. *Ancenay & Maire, "A Time and Energy Optimal Controller for Mobile Robots":* simple discrete motion model, ONLY as example of Quadratic Programming

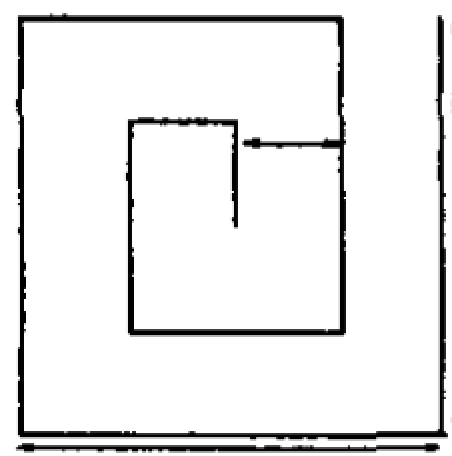
Focus Paper

- Y. Mei, Y.-H. Lu, Y. C. Hu, and C. S. G. Lee. **Energy-Efficient Motion Planning for Mobile Robots**. In *International Conference on Robotics and Automation, pages 4344–4349, 2004.*

- **<u>Problem Statement:</u>**
  - To investigate Energy Efficiency of a mobile robot for Open Area Covering using different methods (namely, Scan-lines, square spirals, and spirals)





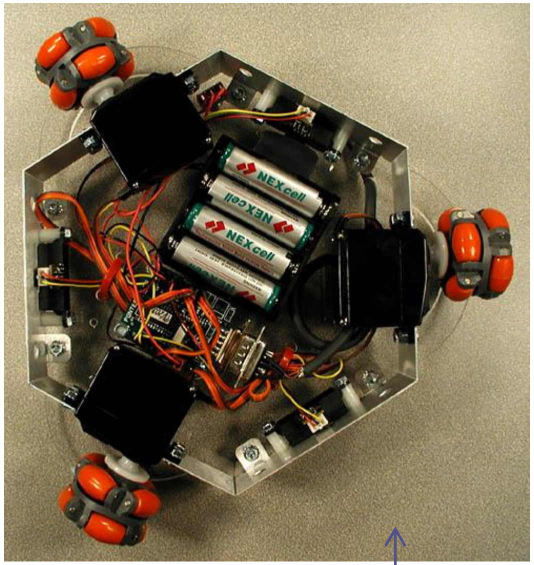


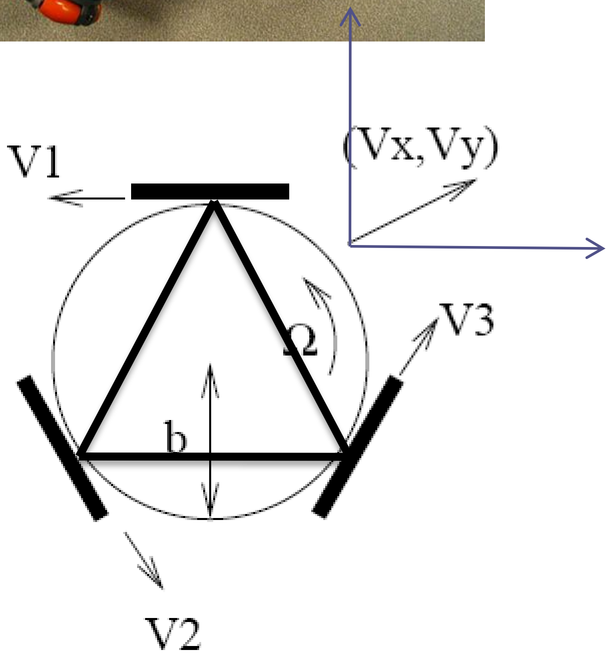
Robot Model

- Palm Pilot Robot Kit (PPRK), CMU

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & b \\ 1/2 & -\sqrt{3}/2 & b \\ 1/2 & \sqrt{3}/2 & b \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ \Omega \end{bmatrix}$$

- Omnidirectional
- 3 servomotors

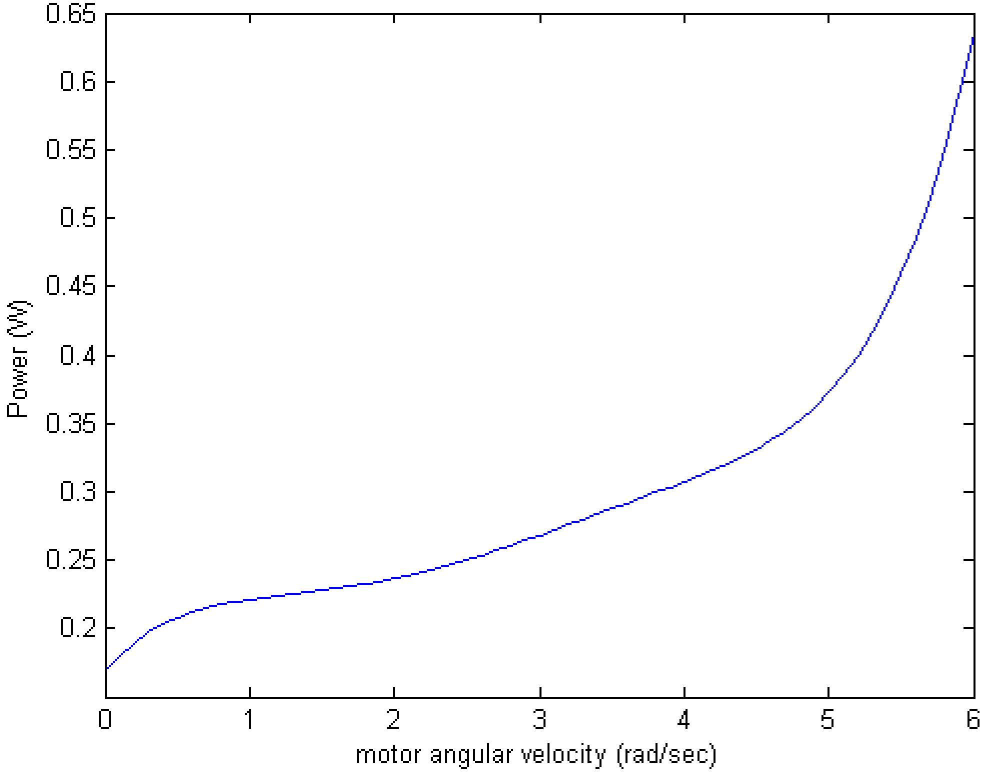


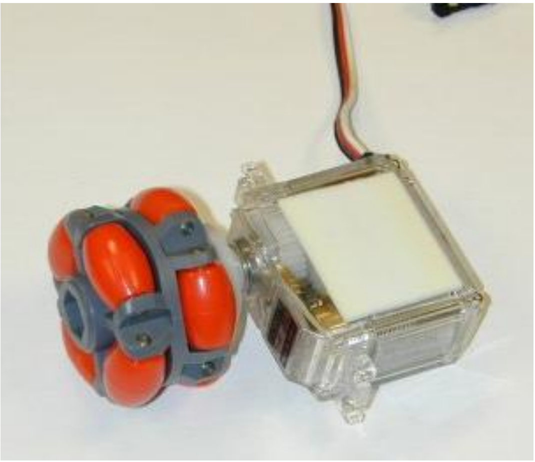


Power Model

- Experimentally,

$$P_m(\omega) = 4.216 \times 10^{-5}\omega^6 + 1.387 \times 10^{-4}\omega^5 - 6.777 \times 10^{-3}\omega^4 + 4.238 \times 10^{-2}\omega^3$$
$$-1.016 \times 10^{-1}\omega^2 + 1.178 \times 10^{-1}\omega + 1.695 \times 10^{-1}.$$





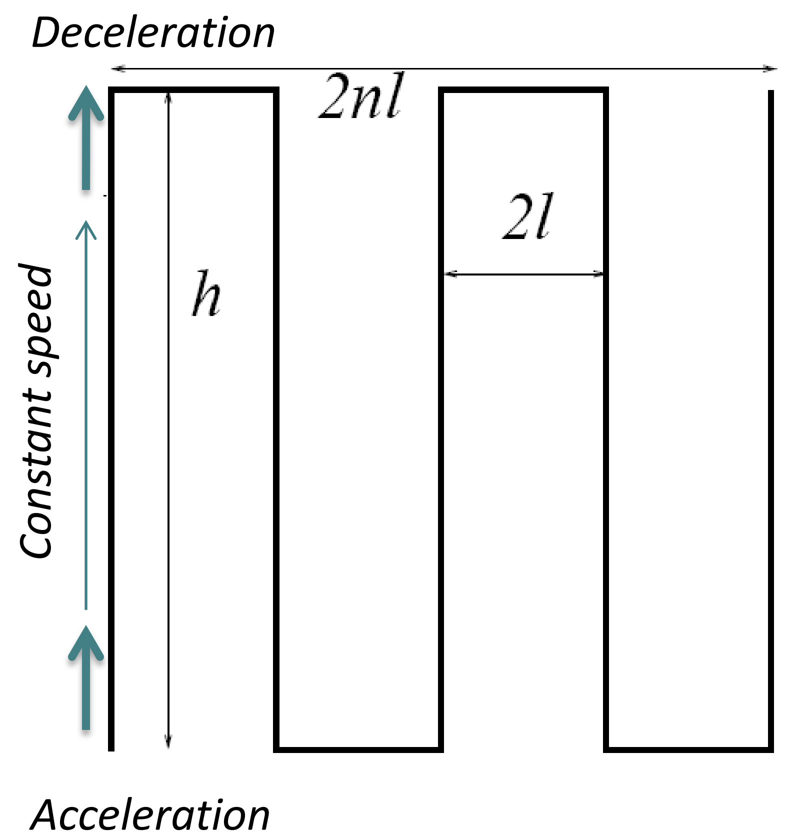
For whole Robot: $$Power(V) = P_m(\frac{V_1}{r}) + P_m(\frac{V_2}{r}) + P_m(\frac{V_3}{r})$$

Scan-lines Area Covering

- At each straight line:
  - Acceleration and deceleration with ‘A’ m/s^2
  - Until constant speed of ‘S’ m/s
- At each corner:
  - Rotation of -90 degrees
  - With angular Acc. & Dec. of ‘Λ’
  - angular speed of ‘Ω’
- At Constant speed:

$$P_S = Power\left(\begin{bmatrix} 0 \\ S \\ 0 \end{bmatrix}\right) \qquad E_1 = P_S \cdot t_S$$

$$t_S = \frac{[(n+1)(h - S^2/A) + n(2l - S^2/A)]}{S}$$



$$Area = (2nl + 2l)(h + 2l)$$

Scan-lines Area Covering

- At accelerations and decelerations:

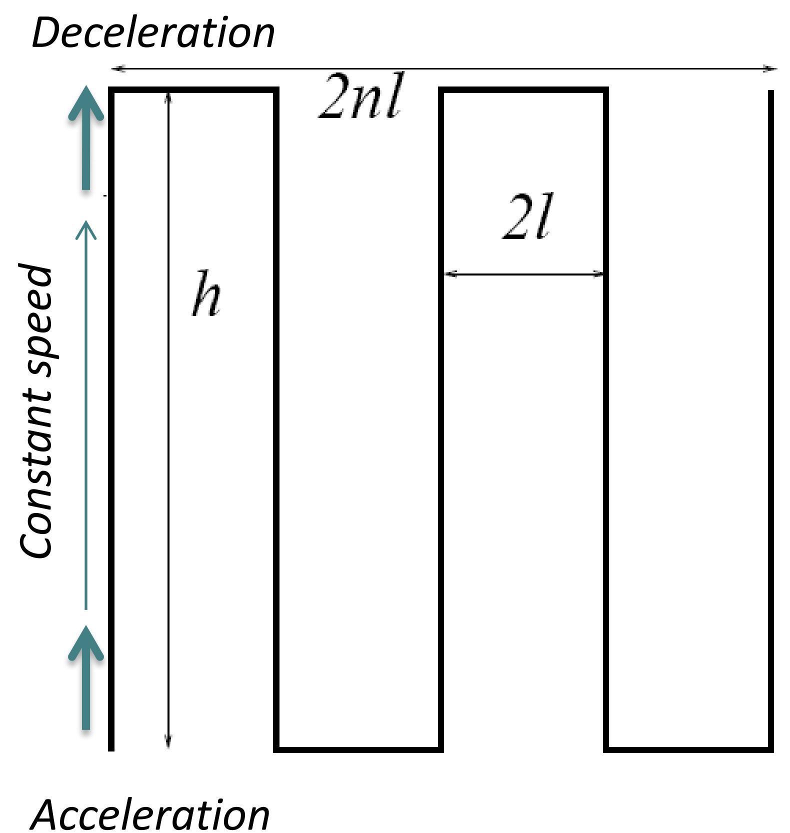
$$P_A = Power(\begin{bmatrix} 0 \\ At \\ 0 \end{bmatrix}) \qquad P_D = Power(\begin{bmatrix} 0 \\ S - At \\ 0 \end{bmatrix})$$

$$E_2 = (2n+1)\cdot[\int_0^{S/A} P_D dt + \int_0^{S/A} P_A dt]$$

- At Rotations:

$$P_\Lambda = Power(\begin{bmatrix} 0 \\ 0 \\ \Lambda t \end{bmatrix}) + Power(\begin{bmatrix} 0 \\ 0 \\ \Omega - \Lambda t \end{bmatrix}) \qquad P_\Omega = Power(\begin{bmatrix} 0 \\ 0 \\ \Omega \end{bmatrix})$$

$$E_3 = 2n \cdot \int_0^{\Omega/\Lambda} P_\Lambda dt + P_\Omega \cdot [\frac{(2n)(90 - \Omega^2/\Lambda)}{\Omega}]$$



$$Area = (2nl + 2l)(h + 2l)$$

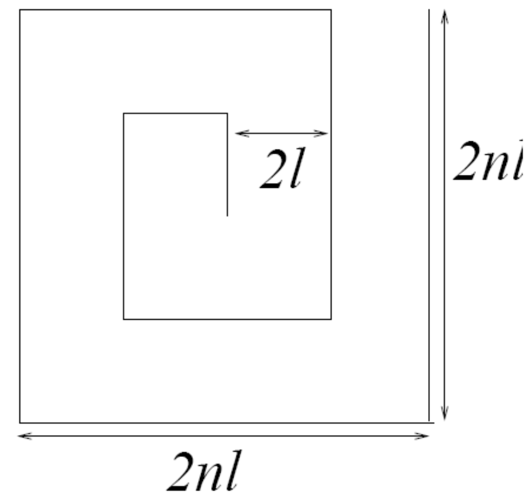
Square Spiral Area Covering

- Rotation are **same** as scan-lines
- Accelerations and deceleration are **same** as scan-lines
- However, for constant speed motion**,**

With: $$E_1 = P_S \cdot t_S$$

$$t_S = \frac{1}{S}[(2nl - S^2/A) + 2\sum_{k=1}^{n}(2kl - S^2/A)]$$

Change in distance

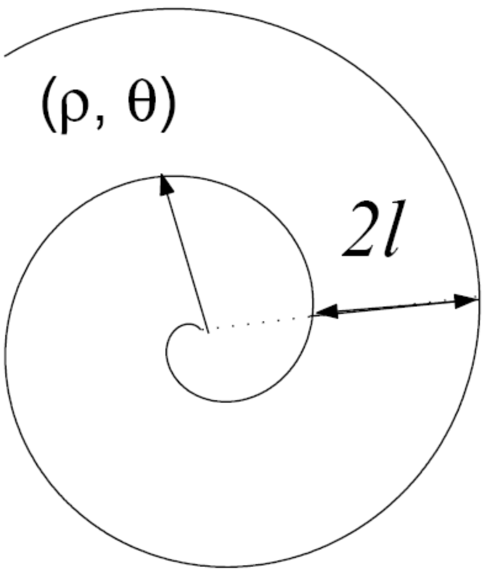


$$Area = (2nl + 2l)^2$$

Spiral Area Covering

- Motion along the curve: $pos = \rho \cdot \theta$
- To have separation of *'2l' every 2π*: $\rho = \frac{2l}{2\pi}\theta = \frac{l}{\pi}\theta$
- Curve position $pos = \frac{l}{\pi}\theta^2$
- For any full rotation of θ_F

$$Area = 2l^2 + \int_{\theta_F - \pi}^{\theta_F} \tfrac{1}{2}(\tfrac{l}{\pi}\theta + l)^2 d\theta = 2l^2 + \tfrac{\pi l^2}{3} + \frac{l^2 \theta_F{}^2}{\pi}$$



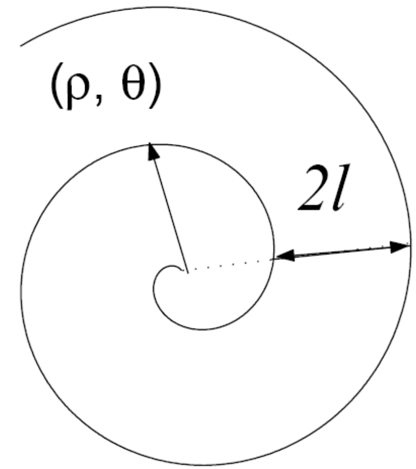
- ***NOTE***: author did not provide much information

along the spiral. Due to the limitation of space, we do not derive the formulas in this paper. Interested readers are encouraged to follow the procedure explained earlier to obtain the analytic form of the energy efficiency.

Spiral Area Covering

- ***All derivation here are done manually***
- curve position: $pos = \rho \cdot \theta$
- So, $x = \rho \cdot \cos\theta$, however: $\rho = \frac{l}{\pi}\theta$

  $y = \rho \cdot \sin\theta$



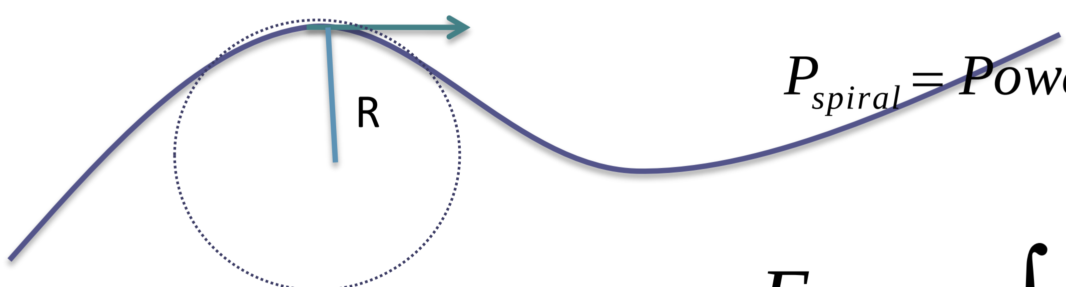
- So, $V_x = \frac{l}{\pi}\dot{\theta}[\cos\theta - \theta\sin\theta]$ we require linear speed ‘S’

  $V_y = \frac{l}{\pi}\dot{\theta}[\sin\theta + \theta\cos\theta]$

- So, $S = \sqrt{V_x^2 + V_y^2} \Rightarrow \dot{\theta} = \frac{\pi \cdot S}{l \cdot \sqrt{1+\theta^2}}$, from this we can have V_x, V_y
- For rotation:

$$R = \frac{1}{K}$$

$$K = \frac{\pi(\pi^2\rho^2 + 2l^2)}{(\pi^2\rho^2 + l^2)^{\frac{3}{2}}}$$



$$P_{spiral} = Power\left(\begin{bmatrix} V_x \\ V_y \\ \Omega \end{bmatrix}\right)$$

$$\Rightarrow \Omega = K \cdot S$$

$$E_{spiral} = \int P_{spiral} dt$$

How did I programmed MATLAB?

$$energy = E = \int Power \quad dt$$

- Programmed function to calculate Power for any robot at any time instant

```
>> rob_power([Vx, Vy, Ω], J, r)
```

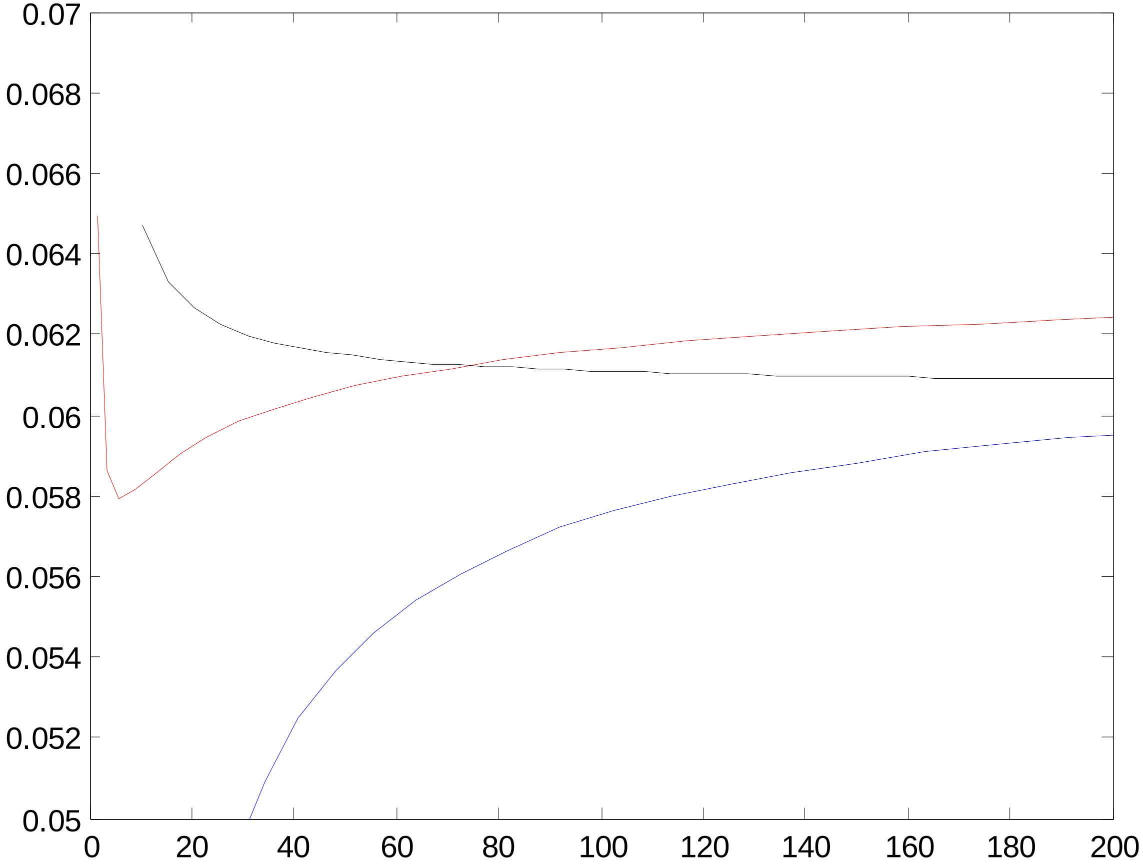
With 'J' is Jacobian of any robot, 'r' wheel radius

- For Scanline & Square spirals:
  - Very simple because of minimal number of variables changing at each ***segment***: either linear speed or rotation speed

- However for Spirals, all 3 variables are changing
  - The integration (numerical) needed some change of variable : $dt = \frac{d\theta}{\dot{\theta}}$

Results: Reproduced

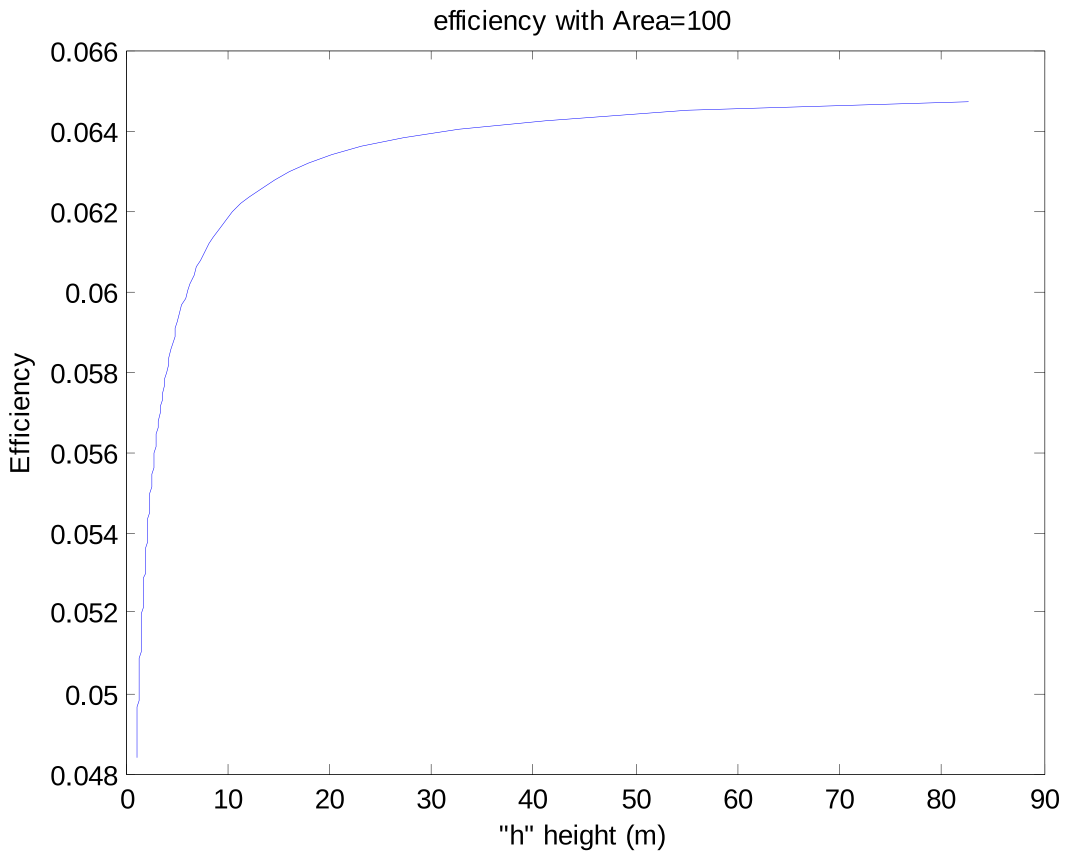
- First for: $S = 0.08 \quad A = 0.2 \quad \Omega = \frac{2}{3} \quad \Lambda = \frac{5}{3}$
- With robot specifications of: $b = 0.12m \quad r = 0.02m$
- And values of: $l = 0.3m \quad h = 8m \quad n$ Varies to control area

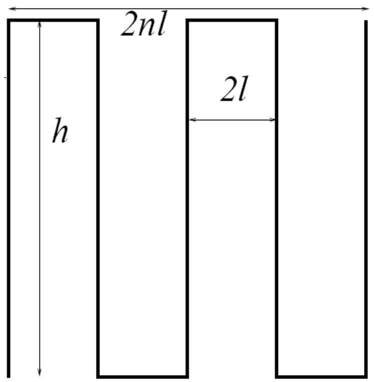


Blue: spiral! (!!)
Red: square spirals
Black: scan-lines

Results: Reproduced

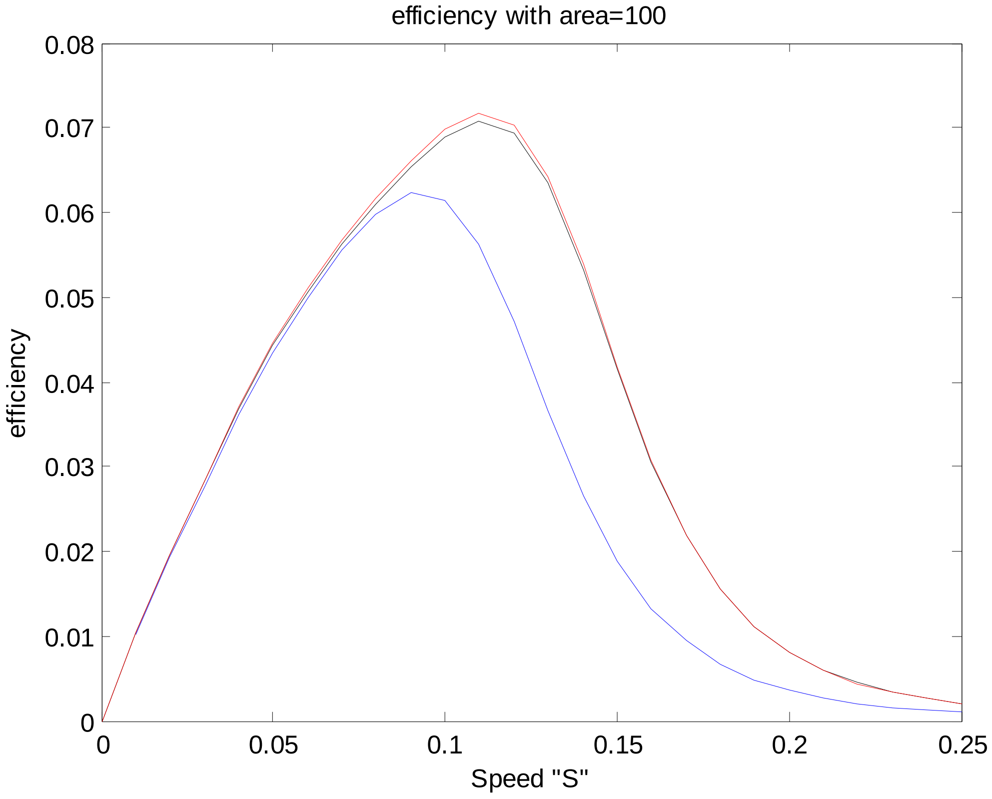
- Scan-lines with area=100, “h” variable

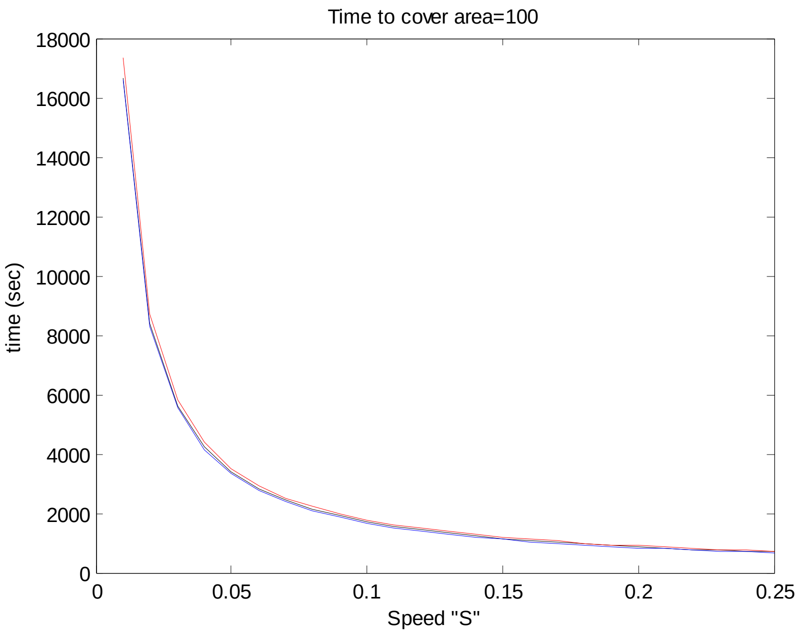




Results: Reproduced

- area=100, “S” variable

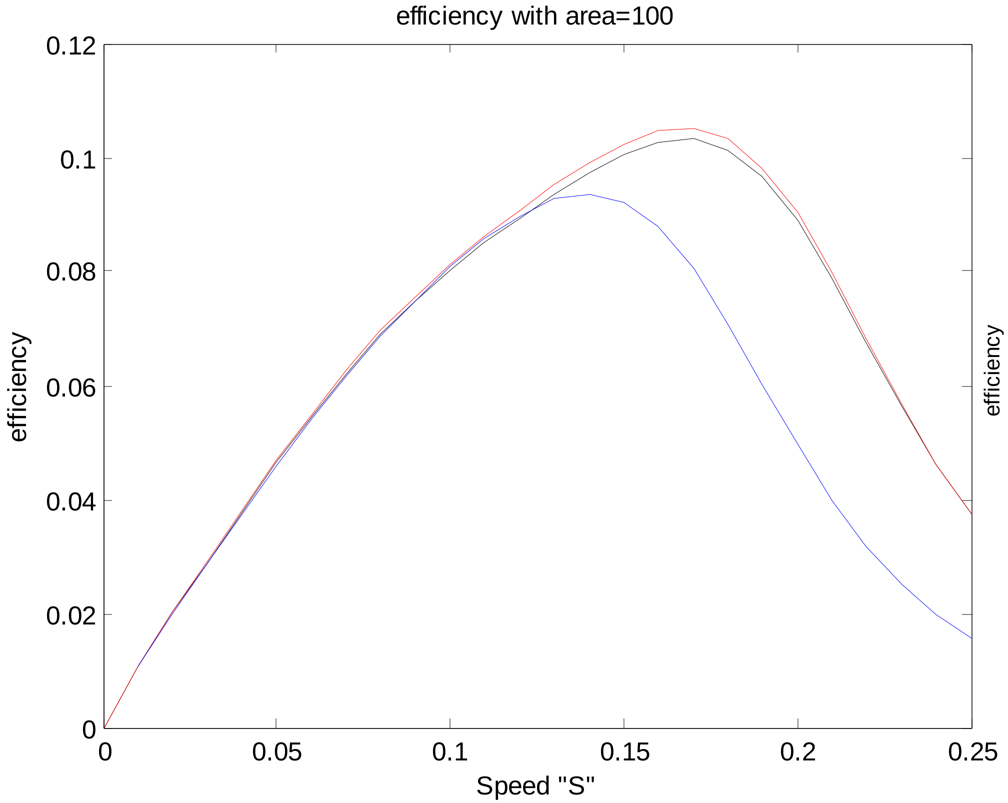


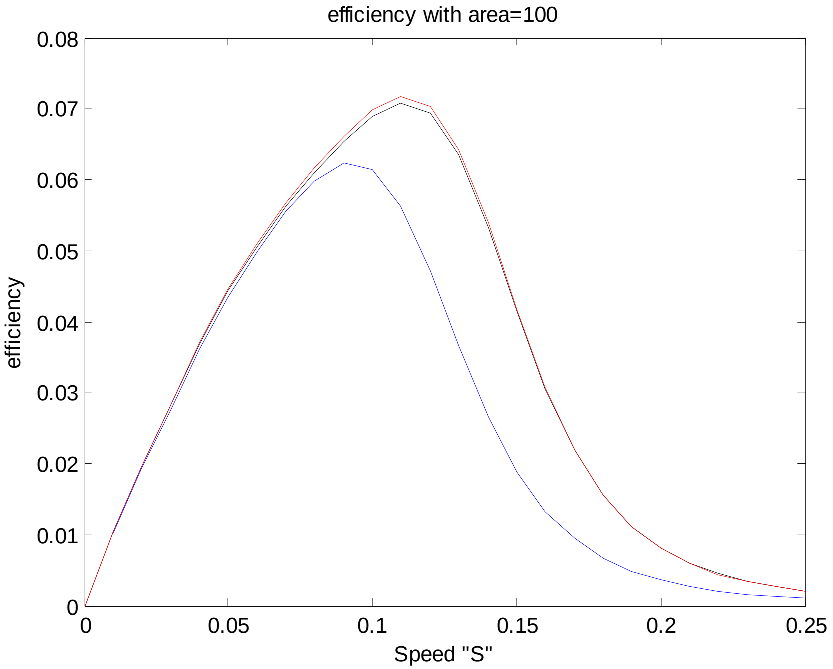


Blue: spiral!
Red: square spirals
Black: scan-lines

Extension: different parameter

- area=100, “S” variable, with ***wheel radius=0.03***



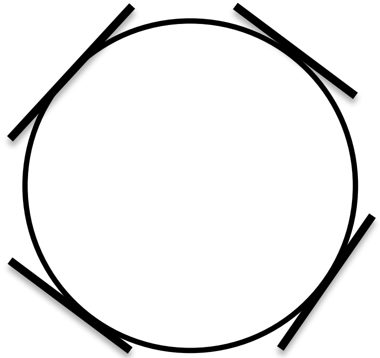


Wheel Radius=0.02

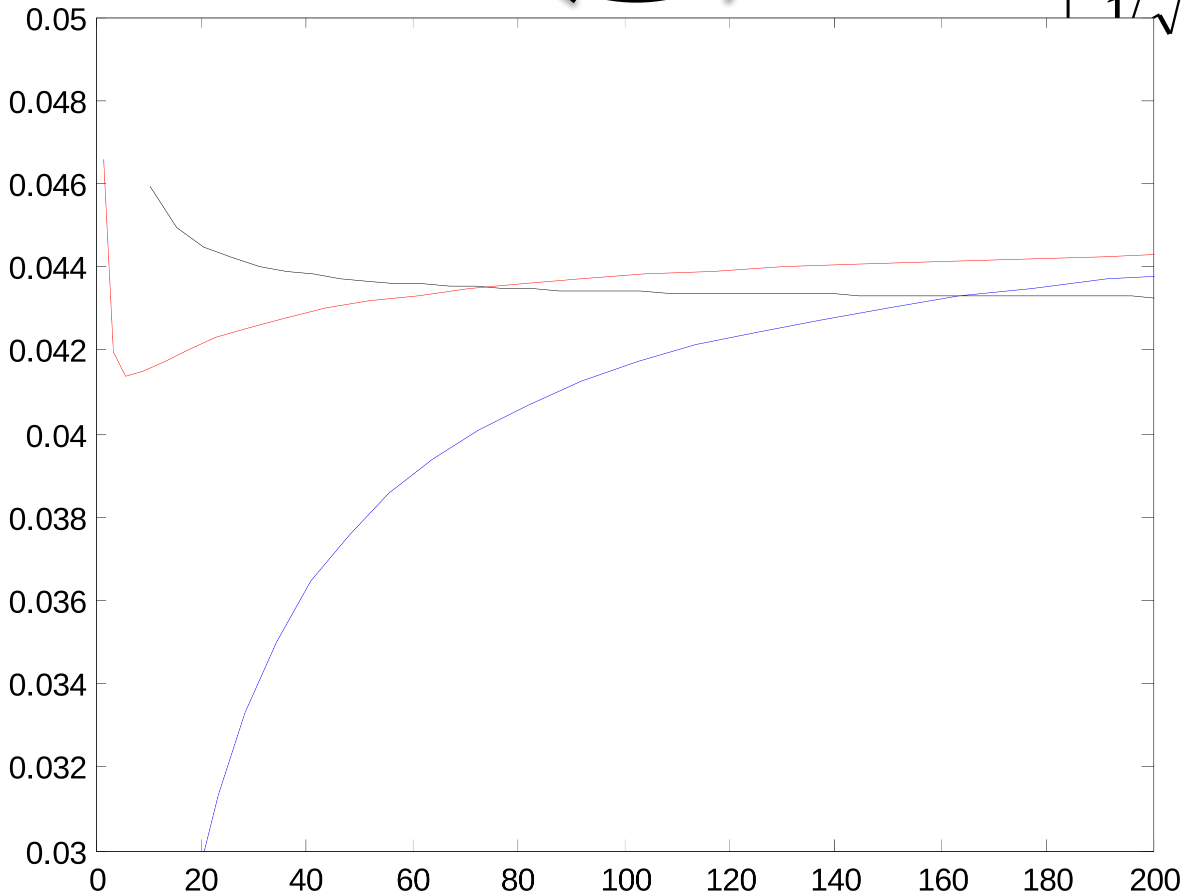
Blue: spiral!
Red: square spirals
Black: scan-lines

Results: Another structure

- For the robot:



$$J = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} & b \\ 1/\sqrt{2} & 1/\sqrt{2} & b \\ -1/\sqrt{2} & -1/\sqrt{2} & b \\ 1/\sqrt{2} & 1/\sqrt{2} & b \end{bmatrix}$$



Blue: spiral! (!!)
Red: square spirals
Black: scan-lines

Future Directions

- **Focus Paper work:** Only predefined areas, Three predefined methods
- An extension is to test for other methods
- A better extension is towards abstraction of the problem:
  - **To plan energy-optimal area covering "path"**
  - **The area could contain obstacles, terrain**
  - **Find a way for optimization given only the power model of the motors**

- **General Comments:**
  - No works for **analytic** solutions of relation between path and energy consumption
  - Most works are experimental power (energy) models
  - Most use only Kinematic Model (steady state). **future**:
    - transient behavior of motors
    - Torques, friction, load torques
  - Generally, no online energy optimization

- **The essential future direction: Theoretical Foundation**

THANKS

- ***Q & A…***