Identification of a Class of Unstable Processes

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***Abstract* — Identification of a practical process, especially if unstable, is challenging as its model is generally stochastic and nonlinear. In this work we consider a class of unstable processes where the model is identified in a closed-loop operating regime. Important issues in identification are addressed, namely: identification scheme, the closed loop identification of unstable plants, choice of sampling period, and constraints on the estimated model parameters. Further the structure of the identified model may not be identical to that of the physical system due to noise artifacts, and inability to capture faster dynamics. Generally least-squares identification is employed to estimate the parameters of the system wherein all the coefficients of numerator and the denominator coefficients of system transfer function are estimated. In many practical system there are constraints on the model parameters. The identified coefficients using the conventional scheme may not obey the constraint. In this work a novel constrained least-squares identification scheme is proposed where in *a priori* known structural constraint is factored in parameter estimation. This scheme is evaluated on a physical magnetic levitation system.**

***Index Terms* — Systems Identification, Closed-loop Identification, Constrained Least-Squares, MAGLEV**

I. Introduction

Identification of a practical process is challenging as its model is generally stochastic, complex and nonlinear [1-6]. The identified model must be simple and linear for designing a controller and further the controller designed using the identified model must meet robustness and performance requirement when implemented on the actual process.

In this work we consider a class of unstable processes where the model is identified in a closed loop operating regime. Important issues in identification are addressed, namely

- Identification scheme
- The closed loop identification of unstable plants
- Choice of sampling period

This paper is organized as follows: Section II gives an overview of the paper, Section III describes the practical evaluation system, and experimental procedure, Section IV discusses the proposed scheme its evaluation, and finally Section V gives conclusion.

II. Overview of Proposed Work

An overview of various issues in identification are investigated, namely, identification scheme, closed-loop identification, and choice of sampling period.

A. Identification Scheme

Generally least-squares identification is employed to estimate the parameters of the system, namely the coefficients of the numerator and the denominator polynomials of the system transfer function. In many practical systems, there is structural constraint on the parameter which is known *a priori* from the physical laws governing the system. Let us consider the following examples:

- In a liquid level system, position control system, aerospace system, the plant may be unstable with a pole at the origin.
- In magnetic levitation system, the there are pair of poles which are located symmetrically about the imaginary axis with one stable pole and the other unstable pole.

As the consequence the parameters denominator coefficients are constrained such that a pole of the system transfer function is at the origin or two poles are located symmetrically about the imaginary axis.

There are two approaches for identification of these systems

- *Unconstrained identification approach*: The *a priori* known constraint on plant parameters is ignored. The conventional recursive least squares method is employed to estimate all the parameters.
- *Constrained identification approach*: The *a priori* known constraints on the parameters are factored in the identification scheme [1]

Using the first approach where no constraint is imposed on the plant parameters, the estimated parameters may not ensure that the estimated model meet the structural constraint because the input may not be sufficiently rich and the output is noisy. In this work, constrained least-squares approach is employed.

B. Closed-loop Identification

Identification of unstable system must be performed in closed-loop operating regime. The parameters of the open loop plant are estimated from the input-output data generated from the plant operating in the closed loop.

As a result of feedback there is a correlation between the input and the output data because of noise input [4]. This correlation may cause in inaccurate parameter estimates when conventional least-squares identification scheme is employed and the input is sufficiently rich.

C. Choice of Sampling Period

The choice of sampling period is crucial in practical system identification as the data is generally noisy and the model is complex and nonlinear. Sampling frequency should be greater than twice the bandwidth for system identification. As faster data acquisition systems are available in recent times, one may be tempted use a very high sampling frequency. Using a very large sampling frequency (compared to twice the bandwidth) may pose the following problems:

- The identified model will capture the noise model as well
- The identified parameter will be such that the poles close the imaginary axis will drift toward the origin. For example in magnetic levitation system the poles on either side of the origin will drift toward the origin where as system with poles close to the imaginary axis will be fused together at the origin.

III. Magnetic Levitation System & Experiment

The proposed scheme was evaluate on a physical system. The physical system was a Feedback® magnetic levitation system (MAGLEV). See the figure 1 below. Identification and control of the magnetic levitation system has been a subject of research in recent times in view of its applications to transportation system, magnetic bearing used to eliminate friction, magnetically levitated micro robot system, magnetic levitation based automotive engine valves [3,7,8].

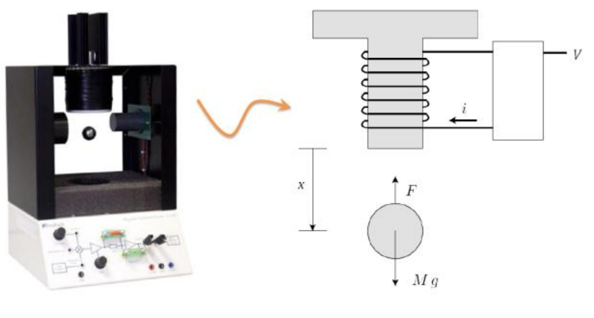


Fig. 1. MAGLEV System and Free-body diagram of the system

The model of the MAGLEV system is unstable and nonlinear [3]

$$m\ddot{x} = mg - \frac{K_c V^2}{x^2} \quad (1)$$

In (1), x is the metal ball position being the system output, V is the system input as the voltage. Other parameters are m as the mass of the metal ball, K_c as constant for magnet circuit, and g is the gravitational acceleration of 9.8 m/s^2. A free-body diagram is shown also in figure 1. We can linearize the system equation at operating point of (V_0, X_0) to have the linear model in (2)

$$\ddot{x} = \frac{2K_C V_0^2}{X_0^3}x - \frac{2K_C V_0}{X_0^2}V$$

$$\Rightarrow \ddot{x} = \alpha x - \beta V \quad (2)$$

So we can have the transfer function for the system as

$$\frac{y(s)}{u(s)} = \frac{\beta}{s^2 - \alpha} \quad (3)$$

Where *y* as the position (output), and *u* as the voltage (input). The poles, *p*, of the plant are real and are symmetrically located about the imaginary axis

$$p = \pm\sqrt{\alpha}$$

Furthermore, we can have the discrete-time model of the system. Eq. (4) shows the discrete transfer function.

$$\frac{y(z)}{u(z)} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (4)$$

With z^{-1} is the unit delay operator in z-domain.

Experiment. We can see that the system we deal with is unstable. So it is expected that the experiment is done while operating on closed-loop. Closed-loop identification is described in figure 2. To identify the plant model, input-output data is processed.

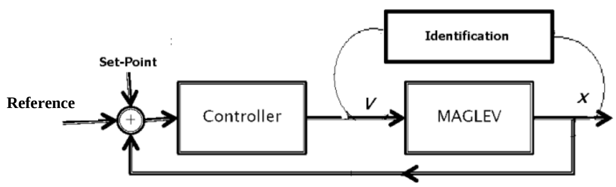


Fig. 2. Closed-loop Identification

Model of the system was identified in closed-loop using data acquired by National Instrument's DAQ devices. The analog controller on-board is a lead compensator. In order to excite all modes of the system, a rich (in frequency) reference input should be applied to the system [2]. The reference input was chosen to be a rich probing signal, specifically *random binary sequence*. The set-point in our experiment is -1. Figures 3 & 4 below show the output & input data, respectively. Experiment is done to collect 45,000 data points. An appropriate sampling period was determined by analyzing the input-output data for different choices of the sampling periods. See figure 5 for partial output data for different sampling periods. A sampling period of 5 ms was found to be the best. The sampling frequency must be sufficiently small to capture the dynamics of the system and not the noise artifacts.

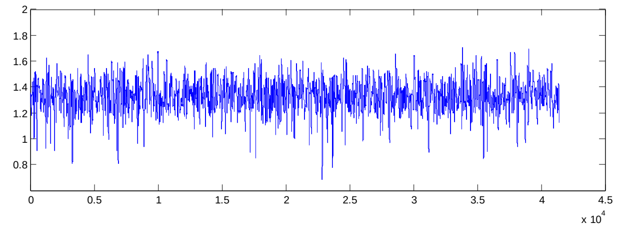


Fig. 3. Output (position) data

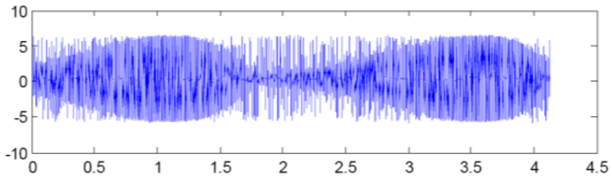


Fig. 4. Input (voltage) data

IV. Identification Process

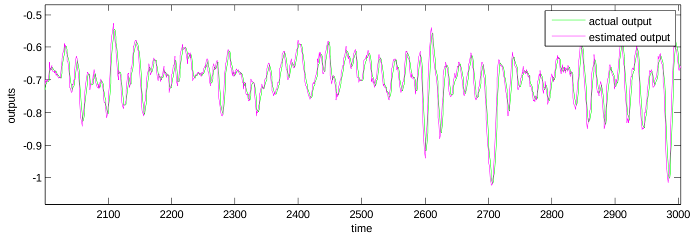
First, we apply the identification in the conventional way. With an ARMA model of

$$y(k) + a_1 y(k-1) + a_2 y(k-2) = b_1 u(k-1) + b_2 u(k-2) \quad (5)$$

You can see that (5) is the inverse Z-transform of (4). With available data, conventional least-squares method is used to find the values of parameters of a_1, a_2, b_1, and b_2. The result of the identification was found to be

$$\frac{y(z)}{u(z)} = \frac{-2.9\times10^{-6} z^{-1} + 0.0001233\, z^{-2}}{1 - 1.837 z^{-1} + 0.8374 z^{-2}} \quad (6)$$

You can see in figure 6, that the estimated output matches the actual output. You can see that the poles of the system are not as expected, as at least one discrete



pole should appear outside the unit circle.

Fig. 6. Estimated (magenta) and Actual (green) output with conventional least-squares methods.

In this work, a novel identification scheme using *a priori* known structural constraint on system model is proposed. This scheme is evaluated on the MAGLEV system. We formulate the problem as a Nonlinear

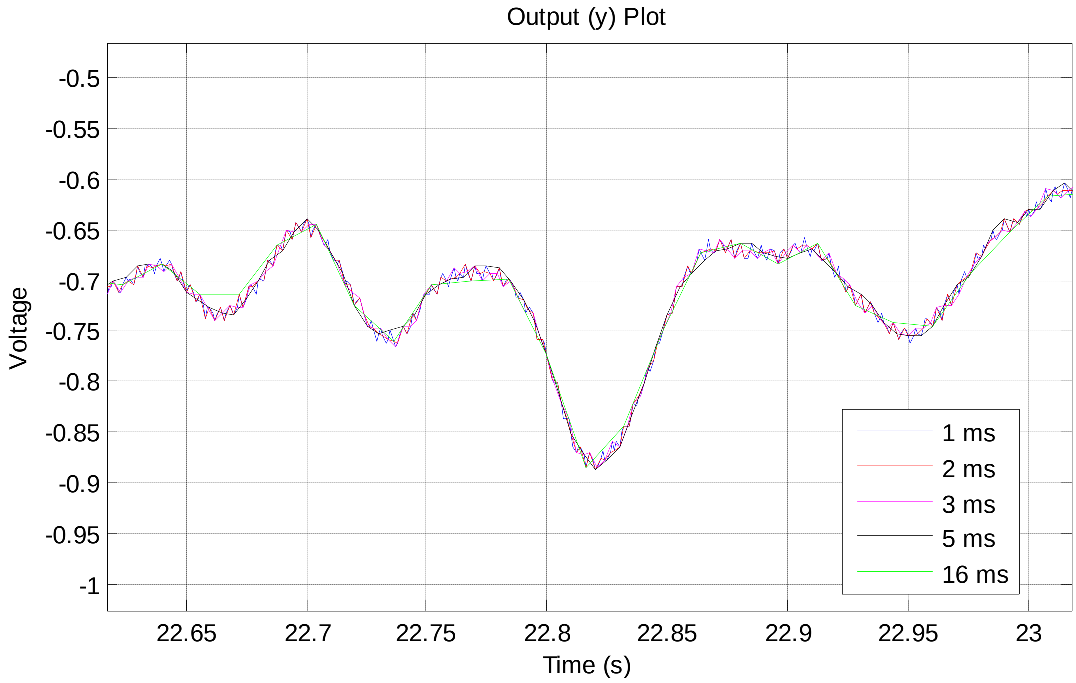


Fig. 5. Output data (partial) with different sampling periods

Optimization problem. See eq. (7) as it shows the complete mathematical formulation of the constrained least-squares method.

$$\begin{aligned} &\min_{\theta} \|Y - A\theta\|^2 \\ &\quad s.t. \\ &f(\theta) \geq 0 \\ &g(\theta) = 0 \end{aligned} \tag{7}$$

With Y as the vector of all output data, A is the data matrix and θ is the unknown parameters vector. You can see that the optimization problem is subject any *a priori* information of the system translated into functions of $f(\theta)$, and $g(\theta)$ as inequality and/or equality constraints, respectively. In the specific case of the maglev system, we have the information of the behavior of the poles, namely,

$$a_2 = 1, \quad |a_1| > 2$$

The above constraints describe two reciprocal discrete poles (1st constraint), with one of them is unstable (2nd constraint). The optimization problem is solved numerically. It is been found that for different data portions the poles of the system have almost the same result. The resulted system model is found to be of the form of

$$\frac{y(z)}{u(z)} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 - 2z^{-1} + z^{-2}} \tag{8}$$

Figure 7 shows an example of the estimated and actual output by the constrained least-squares.

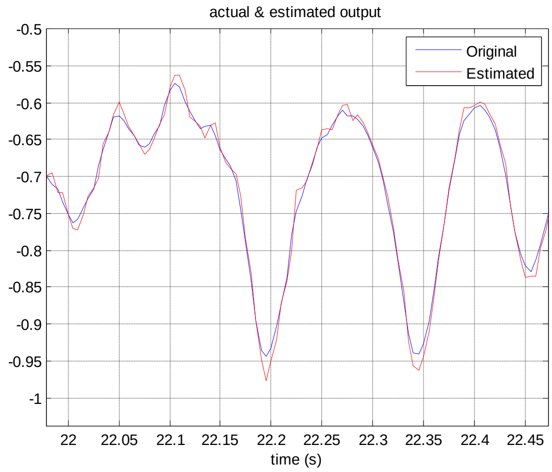


Fig. 7. Estimated (red) and Actual (blue) output with constrained least-squares method.

V. Conclusion

The proposed scheme based on constrained least-squares approach is promising. The result obtained using unconstrained approach was unsatisfactory as the estimated model did not meet the structural constraint. The choice of sampling period is crucial to identification. It is shown that the sampling frequency must be small enough to capture the dynamical behavior and not capture the noise artifacts. The plots of the actual system and the identified model were close.

Acknowledgement

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