The Ideas of Unified Theories of Physics

Prepared by
Tareq Ahmed Mokhiemer
Research Assistant, Physics Department

Introduction

In the current state of physics, all the interactions in the observable world are described by only four forces (i.e., the gravitational force, the electromagnetic force, the weak force, the strong force). It is and it has been intriguing to physicists that we have different force and not only one force. Besides the beauty of physics in case it has been achieved to describe every thing by a single theory, there are some strong evidences that at some certain energy scale, all the four (or at least three) forces merge together in a single force. After the standard model has been stabilized by the unification of the weak theory and the electromagnetic force in the electroweak theory, it has been the goal of many physicists to reach a theory of everything, an economic, simple theory that works at all energies, at all scales, for all time. This appears in the variety of new theories that emerged in the last three decades such as string theory, supersymmetry, quantum gravity, superstring theory, M theory…… Just go to Google and search for “final theory” AND “physics” and enjoy reading through more than 70,000 web pages that talk about the dream of the final theory in physics!

The intuitive idea that all the physical theories are only different faces of a single pattern that repeats itself at different situations in an apparently different fashions has many evidences. In my opinion the fact that there is only One God in the universe and that Allah has set certain laws and rules that doesn’t change or break in many aspects of life gives an indication that all the interactions in the universe is a manifestation of some single concept that governs all of them. Moreover in the currently accepted model of the universe, it’s thought that at the very early stages of the universe, there was only one force in action.

A very striking evident that a single theory should exist that combines the all the existing forces is that when extrapolating the strengths of the three forces in the standard model up to very high levels of energies we notice that all the forces meet at a single point. As shown in figure (1) at 10^{17}GeV, the strong, electromagnetic and gravitational forces meet together.

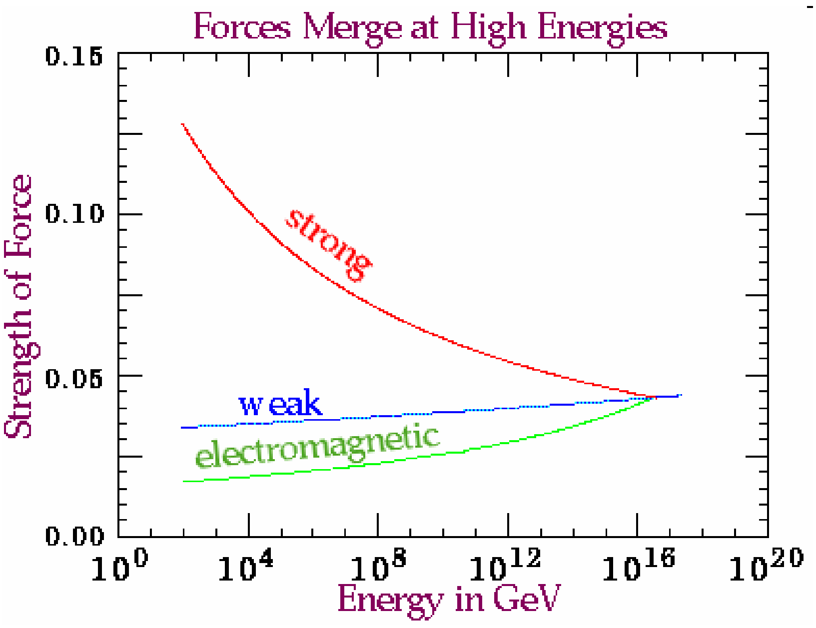

Figure (1)

What's missing in the standard model?

Deep inside the Standard Model, physicists think, something is wrong. There must be a larger, more elegant theory, a theory of everything. Though the standard model is the currently most accepted theory of physics, it's, as physicists think, full of gaps and unanswered questions. The trials to fill these gaps constitute the branch of physics known as "Beyond the standard model". An important part of the physics beyond the standard model is the theories that try to unify the remaining two other forces to the electroweak theory, namely gravity and the strong force. So the first flaw within the standard model is that it doesn't answer the question why do we have four forces and not one force? In the standard model the charge is quantized while the mass is not. There is no explanation for that. Many constants (upto 22 constants) exist in the standard model which are included in it by hand (for example the masses of the fermions and bosons, the coupling constants and the coeffcients of the CKM-matrix all are measured through experiments) . This is thought to be a very large number for a theory that is considered the most accepted theory in physics. The Higgs boson which is considered an essential part of the electroweak theory that is responsible for giving the W, Z bosons their masses hasn't been discovered yet. In fact the Higgs field itself was inserted into the standard model by hand to account for the non-zero masses of these bosons. There is no explanation why the parity and other fundamental symmetries are violated in the weak force alone. Additional anomalies within the standard model are there. For example, in the standard model the neutrinos have zero masses, while recent experiments show that they may have nonzero masses. The anomalous magnetic moment of the pion, the spin crisis of the proton, are other examples. For all these reasons, theorists are trying to go beyond the standard model.

Symmetries and Yang-Mills theory

Symmetries play a great role in contemporary physics. Previously, the laws of physics were anticipated from experimental facts, and then sometime later someone observes that the lawas contain some sort of symmetry. The most evident example for that is Maxwell's equations. The equations were put first by Maxwell's in the nineteenth century. Many years after, it was observed that the laws are invariant under gauge transformation and Lorentz transformation. So symmetry wasn't of fundamental importance prior to the nineteenth century. This situation changed completely in the twentieth century. The great achievement of Einstein was to consider symmetry first, as a primary feature of nature and then from this concept Lorentz invariance was derived. This is a profound change in attitude. Indeed the cornerstone theories of the standard model are built on inherent symmetries and thus the search for further unification of the forces means to search for higher and more profound symmetries of the world. Symmetry means the invariance of the system or the laws under the operation of some transformation in the system parameters. The symmetry may be a global symmetry or a local symmetry. In the former case, the transformation operation is the same in all space-time points. While in the local symmetry, the transformation relation though having the same form at all space-time points, it depends parametrically on the coordinate of spase-time. Of course the local symmetry involves a far greater degree of symmetry than global symmetry. Just from the requirement that the laws of physics are invariant under some local symmetry, all the fundamental laws of physics and even the characters of the elementary particles can be derived. This is the gauge principle. The symmetry underlying the gauge principle may be an internal symmetry like the symmetry in the Schrodinger's equation under the change of the phase of the wave function or a dynamic symmetry like the symmetry under local changes in space-time grid. The former example gives rise to the electromagnetic theory and the later to the general relativity field equations. The first example will be illustrated in more detail later. Associated with each symmetry in nature, according to Noether's theorem, is a conserved quantity. The vice versa is true. In the same time the symmetry leads to the existence of a gauge field that interacts with the conserved quantity. The conserved quantity is the source for the gauge field. The symmetry in question is the symmetry of the Lagrangian itself. Since the equations of motion of a physical theory are determined from the Lagrangian alone, the physics of the system in question will also be symmetric. The only restriction on the symmetry is that it's a continous symmetry, not a discrete symmetry, like the parity. The relations between the symmetry and the gauge field and the conserved quantity is summarized in figure (2).

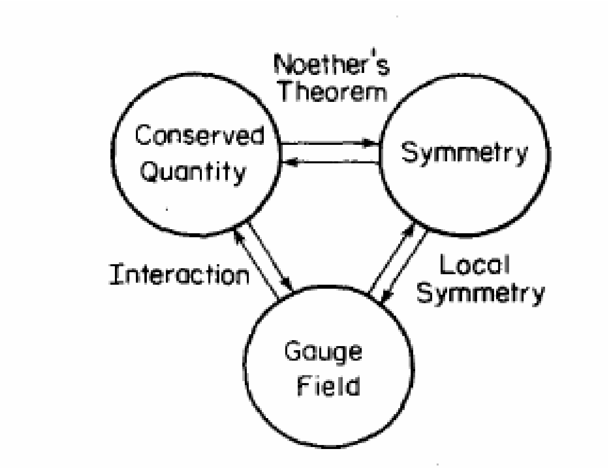

Figure (2)

The three most important theories that constitute physics today, namely general relativity, electroweak theory, and quantum chromodynamics, are gauge theories. Though they are independent, most physicists think that full unification is a matter of time.[1] Thus it's not strange that any new theory that tries to unify these three theories together is supposed to represent a great and natural symmetry of the universe that breaks down under spontaneous symmetry breaking into different sub-symmetries. In other words, the theory should contain a single multi-component gauge field by which all elementary particles can be described. The familiar examples of gauge theories are the electromagnetic theory and general relativity. In what follows, I'll try to give a brief description of the gauge invariance embedded in the electromagnetic theory.

The electromagnetic theory as a gauge theory

Although the invariance of Maxwell's equations under the addition of an arbitrary gradient function to the magnetic vector potential or the time derivative of the same function to the electric potential was known long time ago, the realization that this represents a type of gauge theory in the sense described above had to wait many years later. It was Weyl who first tried to describe electromagnetism as a type of local type of transformation inspired by the success of Einstein's general relativity. The transformation that Weyl tried to use was the invariance under changes of scale. He required that the physics would be the same if each point in space time had its own length scale. This is the origin of the name gauge theory. Einstein pointed out serious flaws in the idea and it was neglected. After the discovery of Schrodinger's equation in 1927, the idea was revived by London who noticed that the invariance of the quantum theory under random changes in the phase of the wave function. To let the symmetry associated with the phase of the wave function be local the change of the phase should be any function of the space coordinates, $\alpha(x)$. i.e,

$$\Psi(x) \;\longrightarrow\; e^{i\alpha(x)}\;\Psi(x)$$

To make the Lagrangian invariant under this transformation we find that the normal partial derivatives in the lagrangian will not be invarant since they will contain a term $\frac{\partial\alpha(x)}{\partial x}$. i.e,

$$\frac{\partial}{\partial x}\Psi(x) \rightarrow e^{i\alpha(x)}\frac{\partial}{\partial x}\Psi(x) + e^{i\alpha(x)}\, i\,\frac{\partial\alpha}{\partial x}\,\Psi(x)$$

To cancel out the effect of this term, we have to modify the form of the partial derivative and replace it by the covariant derivative:. $\frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x} + i\,A_x(x)$

The condition on the vector $\vec{A}(x)$ is that it transforms under the relation:

$$\vec{A}(x) \longrightarrow \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x)$$

We see that this is the same form of transformation in gauge symmetry of electrodynamics. In this case the covariant derivative will transform into

$(\nabla + iq\vec{A})\psi \longrightarrow e^{iq\alpha(x)}(\nabla + iq\vec{A})\psi$ And thus the quantity $\psi^*(\nabla + iq\vec{A})\psi$ is invariant under local phase transformation of the wave function. We notice that the Lagrangian describing the system will also be invariant under such a transformation. And since the equations of motion are determined uniquely from the Lagrangian, the all the physics will be invariant. This is a kind of internal symmetry. For the case of relativistic electrons, the wavefunction Ψ is replaced by the four component Dirac spinors, and following the same arguments of thought, we find the we have to use the covariant four vector derivative $D_\mu = (\partial_\mu - ieqA_\mu)$ to make the Lagrangian take the form $L = \overline{\psi}(i\gamma^\mu D_\mu - m)\psi$. The interaction part involving both the spinor and the new four field in this Lagrangian is given by $L_{\text{int}} = -ne\overline{\psi}\gamma^\mu A_\mu\psi$. If wanted to form an invariant quantity under the gauge transformation involving A we find that the only invariant quantity is : $f_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$. This is the element of the electromagnetic field tensor known in electrodynamics. To be able to derive the equation of motion of the field components we have to add a new term to the Lagrangian that describes the Lagrangian density of the field itself. This new quantity should be gauge invariant and should also give the proper equations of motion of the electromsgnetic field (Maxwell's equation.) It's natural to select it to be proportional to $f_{\mu\nu}f^{\mu\nu}$. The actual Lagrangian density used is: $-\frac{1}{4}f_{\mu\nu}f^{\mu\nu}$. **Thus we see that the local symmetry condition can be satisfied only if we introduced an additional field with all the familiar properties of the electromagnetic field and the whole structure of the electromagnetic theory is uniquely determined by the sole requirement of gauge invariance.** A last note here is the *strange* relation between the phase of the wavefunction and the magnetic vector potential. Experimentally this relation is emphasized in the Aharanov-Bohm effect. Further investigation may lead to a deeper insight into the nature of the magnetic field.

The Weak interactions theory differs from the electromagnetic in that the group of symmetry operations in the SU(2) group is non-abelian group (the group members don't commute with each other). The field resulting of gauging the SU(2) symmetry will be of a different character than the electromagnetic field. Unlike the electromagnetic field, the quant of the new field will carry the source charge of the field (the isospin). The conserved quantity in the weak interaction is the weak isospin, an analogous quantity to the isospin of the strong force. Weak isospin of all fermions has the value of ½ with different projections for different fermions $(\pm\frac{1}{2})$ Thus there are three types of gauge bosons that mediate the weak force constituting a weak isospin multiplet $I^W=1$, with projections 1,-1,0. These are W^+,W^-,W^0 . The leptons are divided into doublets that behave exactly the same way under weak interactions, i.e., the weak interactions doesn't distinguish between an electron and an electron neutrino. In electroweak theory, the two conserved quantity that play a fundamental rule are the iso-spin and the hypercharge. The two are related to the charge by $Q = I_z^W + \frac{1}{2}Y^W$. The group symmetry associated with the electroweak interaction is SU(2)*U(1). A photon-like particle χ^0 is added to the W Boson triplet to account for the U(1) symmetry. χ^0 and W^0 mix together to form the Z^0 and the photon. The problem is that up to now, the predicted value for the masses of all these bosons are zero while we know that the short range of the weak force necessitates the existence of massive mediators. The solution for this was devised by Higgs who added another two terms to the Lagrangian involving a new field, the Higgs' field taking the form of a doublet : $\Phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$. This field is allowed to interact with leptons. The introduction of the Higgs' field is said to have broken the local gauge symmetry of the Electroweak theory spontaneously. Spontaneous symmetry breaking means that although the Lagrangian is symmetric (corresponding to the symmetry of physics) the ground state is asymmetric. A classical example for that is the Ferro magnet. The ground state corresponds to the state where the spins of all atoms are aligned in some 'preferred direction'. Although the equations describing each atom are symmetric in nature, the state of the whole system is not.[1] So Symmetry is hidden rather than broken. This Higgs' mechanism is thought to be responsible to give all the fermions (and thus all the matter) their masses by interacting with this hypothesized field. It's also responsible to give the masses of the massive gauge bosons: W^+,W^-,Z^0. An analogous example is in superconductivity. The alignment of the phases creates an ordered rigid structures giving rise to rejection of the electro-magnetic field (Meissner effect). The photons inside a superconductor are massive. So**,** Massive gauge quanta are possible when

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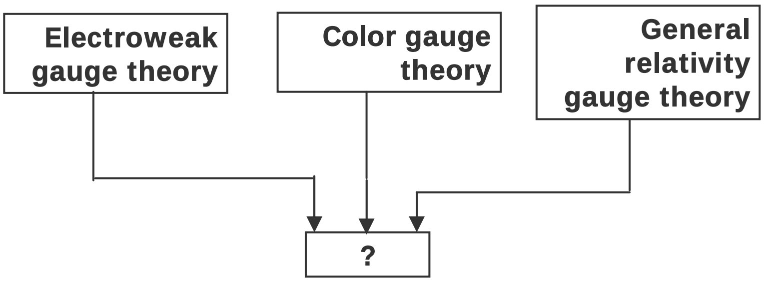
[1] Being the only intrinsic quantity having a physical direction in space, spin may be somehow responsible for all symmetry breaking mechanisms in nature. (mass,charge, parity are scalar quantity. Other quantum numbers, Isospin, has no direction in our physical space, but in another space , isospin space)

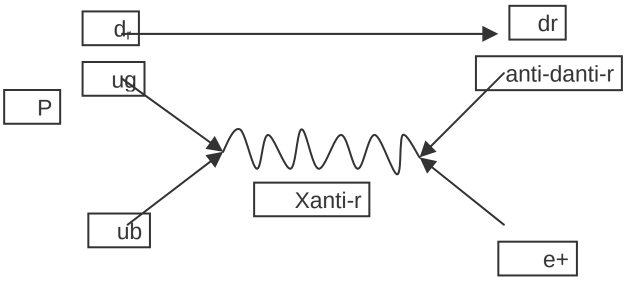
gauge symmetry is broken. Though of such a fundamental importance, the Higg's Boson (the quanta of the Higgs field) hasn't been observed yet. This is considered a flaw in the standard model. It's believed that at the early phases of the universe, all the currently broken symmetries were complete symmetries. And as the universe cooled down, a phase transition happened and these symmetries were broken. The search for new symmetries in the world is based on the search for broken symmetries. If a new symmetry was not broken, it would have been discovered long ago.

General relativity is considerd a gauge theory since the gravitational field equations are invariant under arbitary curvilinear coordinate transformation. So the symmetry here is a dynamical symmetry not an internal symmetry (transformations involving non-dynamical degrees of freedom, flavor and color parameters). The gauge field that has to be introduced to ensure the invariance is the gravitational field. General relativity has not been included in the standard model up to now since its field equations haven't been quantized yet. Another gauge theory is the quantum chromodynamics. The conserved charge in this theory is the color and the group of symmetry is SU(3). So in the current state of physics we have three different and separate gauge theories that haven't been unified yet.



Figure(3)

It's believed that any trials to unify them in a new theory should be built on a new symmetry that includes all of these symmetries. The most promising candidate to unify all these theories is strings theory. Similar to Klein-Kaluza theory, Strings theory add extra dimension to our conventional space time dimensions. The predictions of strings theory require very high energies that are orders of magnitude larger than what the most powerful accelerator can reach nowadays. A less ambitious class of theories are those which try to unify only the electroweak and the strong interactions in a single theory (Grand Unified Theories -GUT). A GUT must include the three group symmetries: SU(3), SU(2), U(1). One trial was conducted by Georgi, and Glashaw in 1975 which is built on SU(5) group. In this theory (and in any other GUT) fermions and quarks become members of the same multiplet to allow them to interact with each other. One direct consequence of such a theory is the non-conservation of the Baryon number. Of course this require the introduction of new and very massive bosons, X and Y bosons, that enable such interaction to happen. One prediction of this theory is the proton decay. A possible Feynman diagram for such a process is shown in figure (4).



Figure(4)

The expected life time for this decay is very large (10^{31}years !). Of course any theory that predicts the decay of a proton should give it a very large life time to account for the stability of the universe. Current searches to detect the very rare event of such a process are conducted in various experiments.

Conclusion

The idea that each of the known elementary particles is a quantum of some multi-component field is very striking. It gives me a feeling that these are just mathematical constructs that had predictions matching the experiments, but they are not the true picture of the world. This seems as if we know only one projection of the reality that we described in a rigorous mathematical way that fits well in our experiments. Physicists are trying to find a single theory that explains the whole universe. My belief is that there will always be some gaps. The part of the universe that we can perceive is very small compared to the part we can not. Cosmological observations (e.g, dark matter, dark energy,…) indicate that we are still crawling under the mountain of a single unified theory

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