

# A STUDY ON THE PERFORMANCE OF SOME ARQ SCHEMES \*

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## Abstract

In conventional radio networks, the data packets (blocks) that are corrupted by errors during the transmission or lost, due to router or receiver limited buffer space, are retransmitted by the source. Usually data blocks contain Error Detection codes to detect whether an error has occurred during packet transmission or not. If an error is detected, the receiver can try to fix the error if the received packet has enough redundant bits, or request a retransmission. In this report, basic Automatic Repeat Request (ARQ) Strategies will be studied over non-fading environment (AWGN) and over Rayleigh fading. The performance of stop and wait, Go-back-N and Selective repeat ARQ will be evaluated. Improvements in the performance will be studied by using variable packet lengths and multi-copy transmissions of negatively acknowledged packets in the case of GBN.

## 1 Introduction

A major concern in data communications systems is how to control transmission errors so that the received data is error free. Communication channels exhibit many kinds of non-ideal behavior, such as additive noise, fading caused by multipath propagation, and intersymbol interference. As a result, the received signal is often badly distorted that the received message cannot be reconstructed unless some kind of error control is used. Basically, there are two approaches to error control in digital communications: forward error correction (FEC) and automatic repeat request (ARQ). In the FEC systems, parity-check bits are added to each transmitted message block to form a codeword. The receiver attempts to locate and correct the errors that it has detected in a received word. After the error-correction procedure, the

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decoded data block is delivered to the end user. A decoding error occurs if the output of the decoder is a different codeword than the one that was originally transmitted. The FEC systems are designed for use in simplex channels. In an ARQ scheme, a high-rate error-detecting code is used together with some retransmission protocol. If the receiver detects errors in the received word, it generates a retransmission request, or a negative acknowledgement (NACK). If no errors are detected in the received word, the receiver sends positive acknowledgements, called an ACK, to the transmitter. cyclic redundancy check (CRC) codes are the most widely used error-detecting codes because of the ease of their implementation. Unlike the FEC systems, the ARQ schemes require the presence of a feedback channel.

In this project, the performance of different ARQ schemes will be simulated and analyzed under erroneous and fading channels. Performance improvements will also be studied by adaptively varying the packet size and sending multiple copies incase of negatively acknowledged packets (GBN) in fading and non fading channels.

## 2 Channel Model

Most of the channel models considered in the literature are discrete-time models, which are characterized by the values of bit error rate (BER) or packet error rate (PER). The simplest discrete-time model is the memoryless binary symmetric channel (BSC) which is also often referred to as the random-error channel. In a BSC, a bit is received in error with a certain probability , independently of all the other bits. The probability of packet error is

$$P_f = 1 - ((1 - P_b)^N) \tag{1}$$

where  $P_f$  is Packet error probability,  $N$  is the total packet size in bits and  $P_b$  is the probability of a single bit error.

In this report, Rayleigh block fading model with BPSK modulation is also used which causes the received signal-to-noise ratio (SNR) to vary randomly.

## 3 Types of ARQ protocols

### 3.1 Stop-and-wait ARQ

In stop-and-wait ARQ, communication is done one frame at a time. After each frame the sender waits for acknowledgement (ACK) signal and doesn't send any further frames until it is received. If the received frame is damaged or lost, the receiver discards it and send a negative acknowledgment (NAK). If a certain time, known as the timeout, passes without ACK or NAK, the sender sends the frame again.

Stop-and-Wait is the simplest in terms of implementation. However, in a real life implementation there are problems to be addressed. The throughput is dependent on the round trip transmission time. This scheme is inefficient as idle time is spent waiting for acknowledgements. Another problem is when the ACK sent by the receiver is damaged or lost. In this case, the sender doesn't receive the ACK, times out, and sends the frame again. Now the receiver has two copies of the same frame, and doesn't know if the second one is a duplicate frame or the next frame of the transmission sequence carrying identical data. Also, when the sender's timeout runs out before the frame reaches the receiver. In this case the sender resends the same packet. Eventually the receiver gets two copies of the same frame, and sends an ACK for each one. The sender, waiting for a single ACK, receives two ACKs, which may cause problems if it assumes that the second ACK is for the next frame in the sequence.

### 3.2 Go-Back N ARQ

Go-back-N ARQ and selective repeat ARQ are more efficient protocols. They assume that multiple frames are in transmission at one time (sliding window). A sliding window protocol allows transmitter to send up to the window size frames before receiving any acknowledgments. When a receiver does acknowledge receipt, the returned pack contains the number of the frame expected next. Using the go-back-N ARQ protocol, if a frame arrives in error, the receiver can ask the transmitter to go back to the  $N_{th}$  frame and retransmit it. After the  $N_{th}$  frame is retransmitted, the sender resends all subsequent frames. The main drawback of this scheme is that whenever an error is detected in a received block, the next N-1 blocks are resent, even though if they were received correctly. These retransmissions are considered as extra overhead and will degrade the overall throughput.

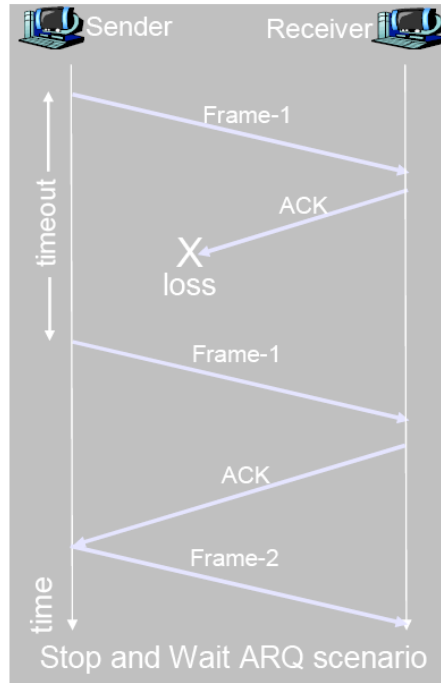


Figure 1: Stop and Wait ARQ protocol

### 3.3 Selective Repeat ARQ

Another variation of ARQ is the selective repeat ARQ protocol. The basic idea behind the selective repeat ARQ protocol is that the receiver accepts packets that are out of order and requests only those frames to be transmitted which are in error. The receiver must have enough buffer space to store the error free packets when an error is detected in a previously received packet.

### 3.4 Type I Hybrid ARQ

Hybrid ARQ is a technique that uses forward error correction (FEC) and error detection coding. In a Type I hybrid ARQ system the message and error detecting parity bits are further encoded with an FEC code. The bits added by the error correction code can be called error correction parity bits to distinguish them from the error detection parity bits. At the receiver, the error correction parity bits are used to correct channel errors. The FEC decoder then outputs an estimate of the message and the error detection parity bits. The FEC decoder output is then tested by the error detection decoder to determine if the message should be accepted as error free, or rejected as containing errors. If the message is long, or if

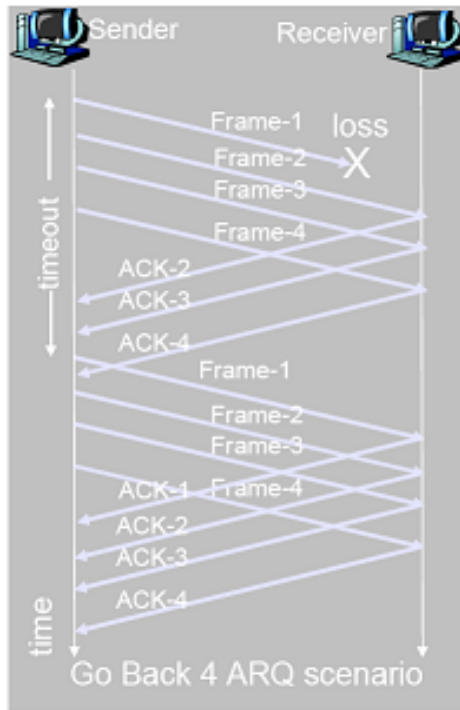


Figure 2: GBN ARQ protocol

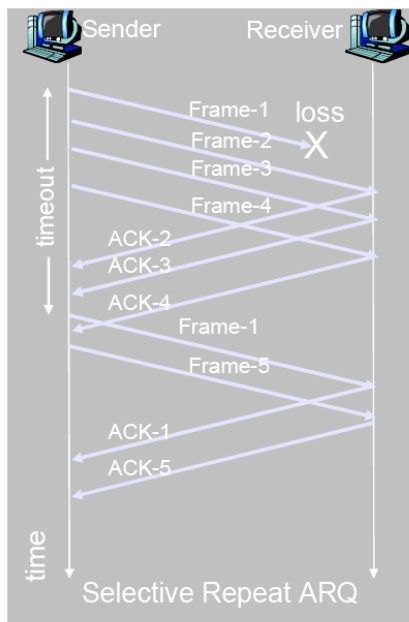


Figure 3: Selective Repeat ARQ protocol

the channel signal strength is poor (high bit error rate), the probability of error-free transmission may approach zero, as shown in the previous section. Under these conditions the efficiency may be improved by using a Type I hybrid ARQ protocol rather than a simple ARQ protocol. This scheme lengthens each transmission with extra FEC parity bits to increase the success probability of each transmission. A coding gain can result if the reduction in the number of transmissions necessary to deliver a message compensates for the increased message length. In strong signal, the Type I hybrid ARQ does not result in an efficiency improvement. When the signal is strong enough to deliver messages error free, then the extra FEC parity bits are wasted. Thus, a crossover point in signal strength exists between plain ARQ and Type I hybrid ARQ as far as efficiency is concerned.

### 3.5 Type II Hybrid ARQ

The adaptively which is desired in time-varying channel environments is achieved to some extent by HARQ-II schemes, where the parity bits for error correction are sent only when they are needed. In a Type II hybrid ARQ scheme the FEC parity bits are not sent with the message and error detecting parity bits. Instead, the message originator alternates between message bits along with error detecting parity bits on one transmission, and only FEC parity bits on the next. On the first transmission attempt, only parity bits for error detection are appended to the message, in the same way as in basic ARQ schemes. If errors are detected in the received word, it is stored in a buffer and a retransmission is requested. The retransmission is not the original codeword but a block of parity-check bits formed based on the original message and an error-correcting code. When this block is received, it is used to correct the errors in the previously stored erroneous word. If the error correction fails, another retransmission is requested, which can be either a repetition of the original codeword or another parity block, depending on the retransmission strategy and the type of error-correcting code that is used. This process continues until the original codeword is delivered successfully. The first point to observe is that when the first transmission is received error free, the FEC parity bits are never sent. Secondly, any error-free copy of the message and the error detecting parity bits delivers the message. Finally, any two consecutive transmissions, if neither is error free, can be combined for error correction, which may successfully deliver the message. The intent of Type II hybrid ARQ is to operate with the efficiency of plain ARQ in good quality signal and obtain the improvement of Type I hybrid ARQ in poor quality signal.

### 3.6 Adaptive ARQ

By an adaptive ARQ scheme, we mean an ARQ scheme with two or more different transmission modes meant for different channel conditions, which uses some channel sensing mechanism to decide which transmission mode is used. A change of transmission mode can mean, for example, a change of the packet size in the SR scheme or a change of the number of transmitted copies of a packet in the GBN scheme or a change of the code rate. In these schemes, the channel sensing is usually done by observing the acknowledgements sent by the receiver to the transmitter.

## 4 Performance Measures

In this section, the performance in non fading channel is derived. Then, these equations can be extended to the case where fading exists.

### 4.1 Efficiency Calculations

#### 4.1.1 Stop and wait ARQ

The effective information rate is given by

$$\begin{aligned} R_{eff} &= \frac{\text{number of information bits delivered to destination}}{\text{total time required to deliver the information bits}} \\ &= \frac{n_f - n_o}{t_o} \end{aligned} \quad (2)$$

where  $t_o$ ,

$$t_o = 2t_{prop} + 2t_{proc} + t_f + t_{ack}$$

is the total delay due to propagation time  $t_{prop}$ , processing time  $t_{proc}$ , frame time  $t_f$  and acknowledgement time  $t_{ack}$ .  $n_f, n_o$  are the size of the total frame in bits and the overhead bits simultaneously.

The transmission efficiency of Stop-and-wait ARQ is given by the ratio  $R_{eff}$  to the data rate R:

$$\eta_o = \frac{\frac{n_f - n_o}{t_o}}{R} \quad (3)$$

When the channel experiences errors, the total time required to deliver a correct frame has to be recalculated. Let  $n_t$  be the number of transmissions required to deliver a frame successfully, then  $n_t = i$  transmissions are required if the first  $i-1$  transmissions are in error and the  $i$ th transmission is error free, thus

$$P[n_t = i] = (1 - P_f)P_f^{i-1}; \quad \text{for } i = 1, 2, 3.. \quad (4)$$

Assuming the timeout period is equal to propagation time,

$$E[t_{SW}] = t_o + \sum_{i=1}^{\infty} (i-1)t_{out}P[n_t = i] \quad (5)$$

$$= t_o + \sum_{i=1}^{\infty} (i-1)t_{out}(1 - P_f)P_f^{i-1} \quad (6)$$

$$= t_o + \frac{t_{out}P_f}{1 - P_f} \quad (7)$$

$$= \frac{t_o}{1 - P_f} \quad (8)$$

Thus, the effective transmission rate for stop and wait ARQ is

$$R_{eff} = \frac{n_f - n_o}{t_o}(1 - P_f)$$

and the transmission efficiency of Stop-and-wait ARQ is

$$\eta_o = \frac{\frac{n_f - n_o}{t_o}}{R}(1 - P_f) \quad (9)$$

In other words, the effect of errors in the transmission channel would reduce the effective transmission rate and efficiency by  $(1 - P_f)$

#### 4.1.2 Go-Back-N ARQ

In this scheme, the inefficiency of SW ARQ can be overcome by keeping the channel busy all the time. The window size  $W_s$  in this scheme is kept larger than the delay bandwidth product so the channel is kept busy all the time.

As before, if  $n_t = i$ , then  $i - 1$  retransmissions of groups  $W_s$  frames are followed by a single transmission of successful frame. The total time required to deliver a frame is



$$E[t_{GBN}] = t_f \left( 1 + W_s \sum_{i=1}^{\infty} (i-1) P[n_t = i] \right) \quad (10)$$

$$= t_f \left( 1 + W_s \sum_{i=1}^{\infty} (i-1) (1 - P_f) P_f^{i-1} \right) \quad (11)$$

$$= t_f \left( 1 + W_s \frac{P_f}{1 - P_f} \right) \quad (12)$$

The effective transmission rate for GBN is

$$R_{eff} = \frac{n_f - n_o}{E[t_{GBN}]} = (1 - P_f) \frac{n_f - n_o}{t_f (1 + (W_s - 1) P_f)}$$

and the transmission efficiency of Stop-and-wait ARQ is

$$\eta_{GBN} = \frac{\frac{n_f - n_o}{E[t_{GBN}]}}{R} = (1 - P_f) \frac{1 - \frac{n_o}{n_f}}{(1 + (W_s - 1) P_f)} \quad (13)$$

#### 4.1.3 Selective Repeat ARQ

If the receiver of the SR scheme has an infinite buffer (i.e. there cannot be buffer overflow), the throughput is independent of the round-trip delay and is calculated as follows:

$$E[t_{SR}] = t_f \left( 1 + \sum_{i=1}^{\infty} (i-1) (1 - P_f) P_f^{i-1} \right) \quad (14)$$

$$= t_f \left( 1 + \frac{P_f}{1 - P_f} \right) \quad (15)$$

The effective transmission rate and efficiency for SR ARQ is

$$R_{eff} = (1 - P_f) 1 - \frac{n_o}{n_f} R$$

$$\eta_{SR} = (1 - P_f) 1 - \frac{n_o}{n_f} \quad (16)$$

## 4.2 Comparison of Stop-and-Wait, Go-Back-N and Selective-Repeat ARQ

For the above 3 schemes, if we neglect the header and CRC overhead, the throughput for the 3 above protocols simplify to

$$\eta_{SR} = \frac{(1 - P_f)}{1 + L} \quad (17)$$

$$\eta_{GBN} = \frac{(1 - P_f)}{1 + LP_f} \quad (18)$$

$$\eta_{SR} = (1 - P_f) \quad (19)$$

Where  $L = 2(t_{prop} + t_{proc})R/n_f$  is the size of the channel in multiples of frames. L is called the delay-bandwidth product.

Figure 4 shows the performance of the three schemes in a non-fading channel. It can be seen that SR-ARQ has the best performance followed by GBN and finally SW ARQ. Simulation of GBN and SW ARQ with different L is shown, and it is shown that as the frame size increases for these 2 schemes, the throughput increases.

## 5 Improving Basic ARQ Schemes

In this section, Selective Repeat ARQ packet size will be varied and multi-copy technique will be used for GBN ARQ.

### 5.1 Selection of the Packet Size

As we have seen, the throughput efficiencies of all the basic ARQ schemes are functions of the packet size N. Consequently, N is a very important design parameter when the performance of the scheme is optimized. There is an obvious trade-off here: for short blocks, the PER is lower and fewer bits are retransmitted in the case of an error; on the other hand, a larger proportion of time is spent on transmitting the actual data bits if longer blocks are used. Packet size optimization for adaptive ARQ schemes has also been

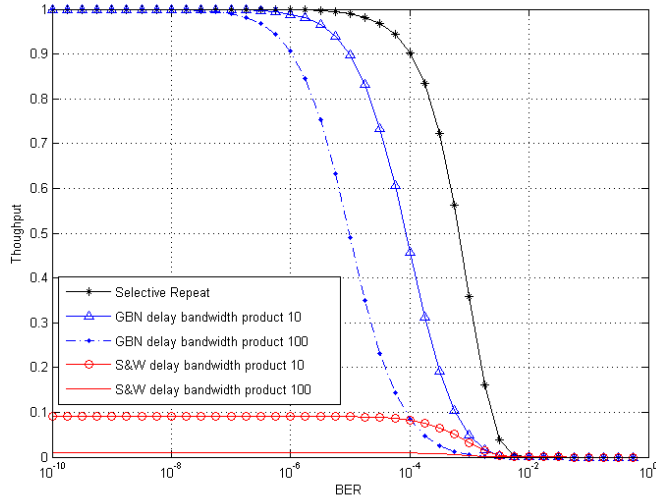


Figure 4: Performance of ARQ protocols as a function of bit error rate for a frame size of 1024 bits and channel delay-bandwidth product of 10 and 100 frames.

considered. To get the optimum packet size for SR-ARQ, equation (16) is differentiated with respect to  $N$  and the derivative is set to zero. Solving for  $N$  will give us the optimum packet size,

$$N_{opt} = \frac{n_o \log(1 - P_b) - \sqrt{n_o^2 [\log(1 - P_b)]^2 - 4n_o \log(1 - P_b)}}{2 \log(1 - P_b)} \quad (20)$$

Figure 5 shows the throughput for SR ARQ for different bit error rates and different packet sizes. If  $n_o = 16$ , the optimal packet sizes corresponding to BER values  $10^{-5}$ ,  $10^{-4}$ ,  $10^{-3}$ ,  $10^{-2}$  are 1273, 408, 135 and 49 bits, respectively. If the transmitter can adaptively set the packet size according to the bit error rate, by measuring the error on the acknowledgments, SR-ARQ throughput can be maximized as shown in figure 6.

## 5.2 Use of Multi-copy Transmissions

Another potential method of improving the performance of the basic ARQ schemes especially in poor channel conditions is to use multicopy transmissions, where multiple copies of each data block are sent contiguously before moving on to the next block in schedule. If at least one of the copies is received successfully, the data block is acknowledged positively. For the SR scheme, the throughput would actually decrease if multicopy transmissions were used because then some successful transmissions would

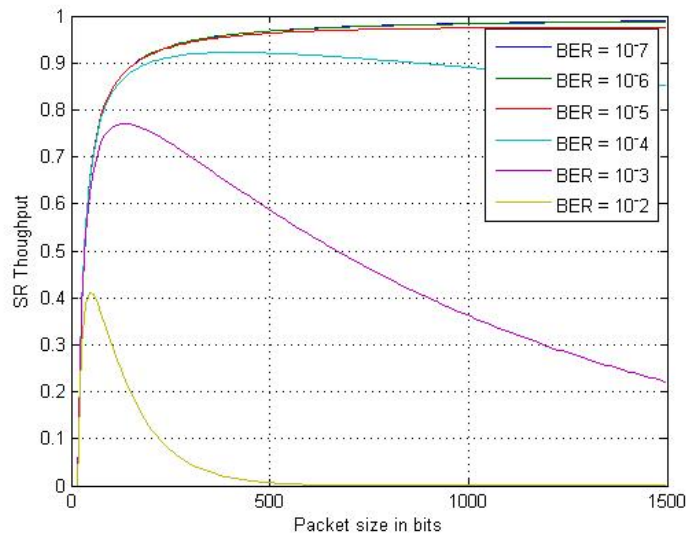


Figure 5: The SR throughput as a function of packet size with six different BER values when  $n_o = 16$  bits.

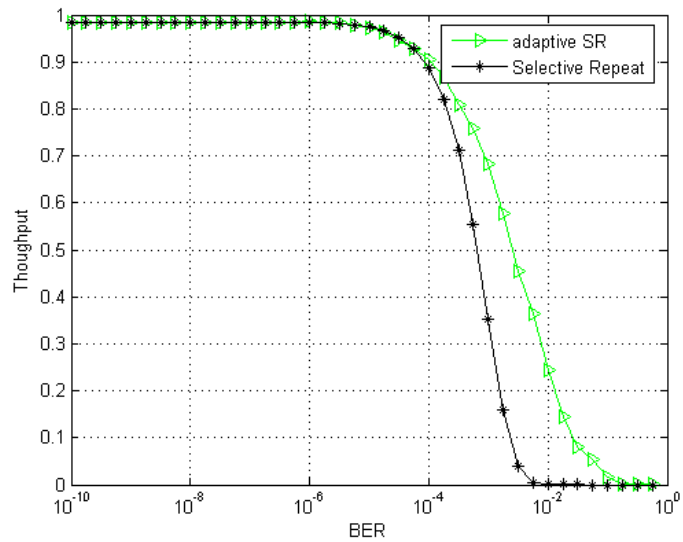


Figure 6: Performance of Adaptive SR ARQ and SR ARQ protocols as a function of bit error rate for a frame size of 1000 bits and  $n_o = 16$  bits.

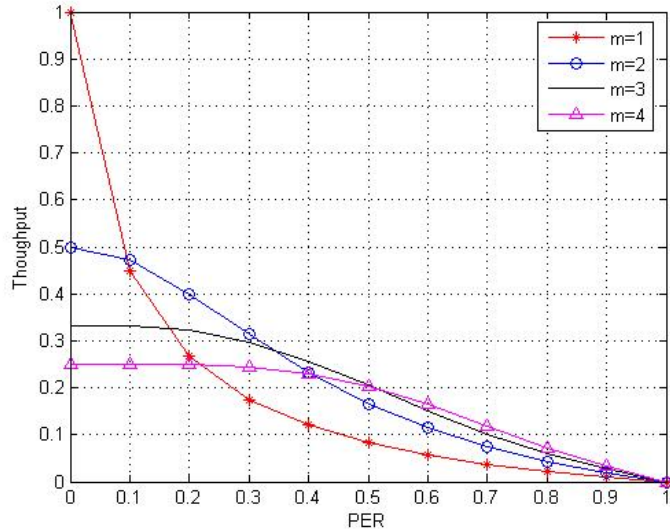


Figure 7: Performance of multi-copy GBN-ARQ protocols as a function of packet error rate,  $L = 10$ .

be wasted, which does not happen in the basic SR scheme. However, if the buffer space at the receiver is severely limited, the probability of buffer overflow can be decreased by sending multiple copies of the packet in the retransmissions. A modified SR scheme based on this idea was proposed and analyzed by Weldon in [6]. In this section, the use of multi copies transmission done by Yao [7] is analyzed for GBN-ARQ. When using  $m$  copies instead of one copy, in equation (18) (assuming overhead is negligible and probability of feed back error is zero), as stated by Yao becomes

$$\eta_{GBN} = \frac{1 - P_f^m}{(m + (LP_f^m))}; \quad , m = 1, 2, 3, \dots \quad (21)$$

Figure 7 plots the throughput of GBN-SR for different  $m$  copies. As shown, when the PER is low, multi copy technique is inefficient to use, never-the-less, when PER is high, it is more efficient.

## 6 Improving Basic ARQ Schemes under Rayleigh Fading

In this section, the throughput of coherent BPSK assuming block fading Rayleigh channel is shown against average SNR using SW-ARQ, GBN-ARQ and SR-ARQ. SR-ARQ is also optimized by using optimum packet length.

The probability of error for coherent BPSK as a function of received average signal to noise ratio (SNR) is given by

$$P(\gamma_b) = Q(\sqrt{2\gamma_b}), \quad \gamma_b = \alpha^2 \frac{E_b}{N_o} \quad (22)$$

The above equation is valid when the attenuation  $\alpha$  is constant, but when  $\alpha$  is random,  $P(\gamma_b)$  must be averaged over the probability density function (pdf) of  $\gamma_b$ , i.e.

$$P_e = \int_0^{\infty} P(\gamma_b)p(\gamma_b) d\gamma_b \quad (23)$$

where  $p(\gamma_b)$  is the pdf of  $\gamma_b$ .

When the attenuation  $\alpha$  is rayleigh distributed,  $\alpha^2$  has a chi-square distribution with 2 degrees of freedom and  $\gamma_b$  also has a chi-square distribution, then the above pdf becomes

$$p(\gamma_b) = \frac{1}{\bar{\gamma}_b} e^{-\frac{\gamma_b}{\bar{\gamma}_b}}, \quad \gamma_b \geq 0 \quad (24)$$

where  $\bar{\gamma}_b = \frac{E_b}{N_o} E(\alpha^2)$  is the average SNR. Substituting the pdf in (24) into (23) and carrying the integration, the probability of symbol error of BPSK becomes

$$P_{BPSK} = \frac{1}{2} \left( 1 - \sqrt{\frac{\bar{\gamma}_b}{1 + \bar{\gamma}_b}} \right) \quad (25)$$

Substituting the above equation (25) in equations (9), (13) and (16), we can evaluate the performance of the different ARQ schemes for coherent BPSK in Rayleigh fading channel.

Figure 8 shows that SR-ARQ has the best performance among SW and GBN ARQ. SR-ARQ can also be optimized by using variable optimum packet length as shown in figure 8. Figure 9 shows the optimum packet length for different SNRs, and it was shown that the optimum packet length was 40, 55, 90, 150, 260 and 550 for an average SNR of 10 dB, 15 dB, 20 dB, 25 dB, 30 dB, and 35 dB respectively.

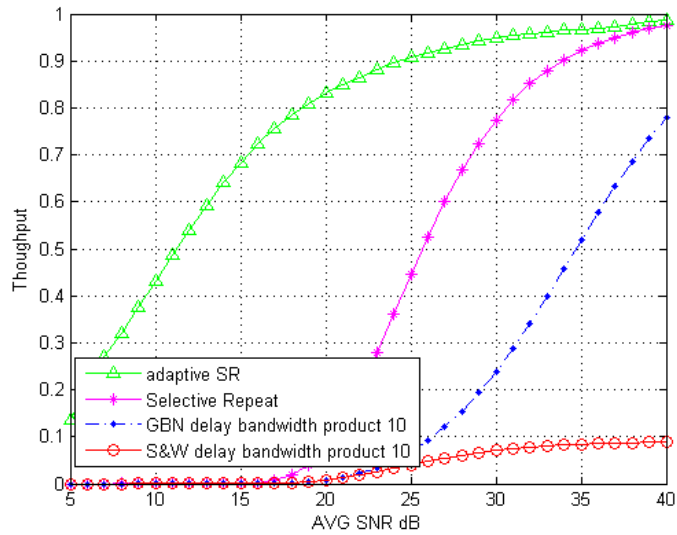


Figure 8: Throughput of SR ARQ system employing coherent BPSK over Rayleigh block fading channel  $N = 1024$ ,  $n_o = 16$  bits.

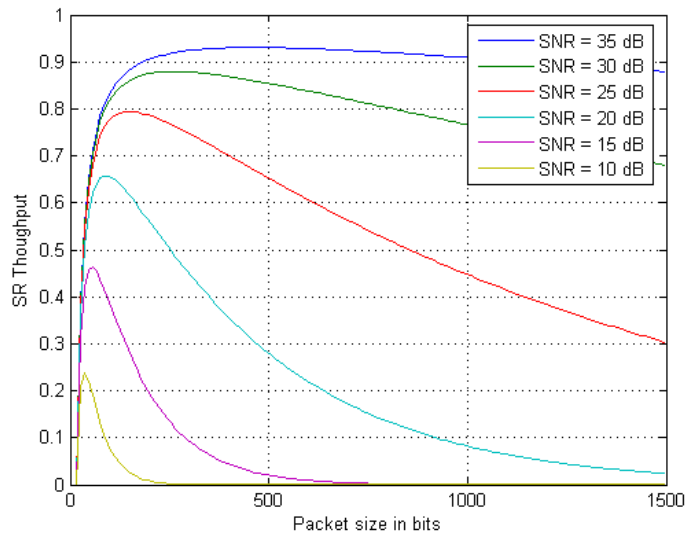


Figure 9: Throughput of SR ARQ system employing coherent BPSK over Rayleigh block fading channel as a function of packet size with six different SNR values,  $n_o = 16$  bits.

## 7 Conclusions

In this report, performance analysis of different ARQ protocols are analyzed in a fading and non-fading channel. It was shown that SR-ARQ has the best performance in both cases. It was also shown that if we increase the packet size for SW and GBN ARQ schemes, the throughput would also slightly increase. Techniques to improve the performance such as adaptive veritable lengths and multi-copies are shown. It was shown that SR-ARQ performs better with variable length packet size. It was also shown that the performance of GBN-ARQ improves with multi-copy transmission when the channel error rate is high, while degrades when the channel bit error rate is high.



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